Fractal Time

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Preface

This work deals with an age-old problem of mankind in a highly modern original way. The geniuses of Aristotle and Mandelbrot are brought together for the first time. Few living scientists are in a position to undertake or venture upon such an attempt.

The work is timeless. Even in a 100 years, it will still be characterised as difficult - despite the fact that it is written in an extraordinarily clear manner. One is reminded of Fichte's "Sonnenklarer Bericht", which is still, even today, characterised as obscure despite its transparency.

The fundamentalidea of the work is a definition of duration which is based on content and may therefore be described objectively, independent of the momentary experience. Mozart's draft of a complete time contour before its first internal or external hearing was a determining motivation. Husserl's and Bieri's ideas and perceptions have been assimilated. If an experience becomes or may become ever richer with every new contemplation of it, this fact reveals something about the structure of time. The duration of tedium and its opposite become, in principle, formally capable of being grasped. Learning is seeing anew, is work on the past.

These are convincing and, within the context of modern theory of the brain, unknown, insights. The may well gain neurobiological relevance.What is most amazing, however, is that Ms Vrobel succeeds in combining her own intuitive approach to the problem of time - an approach which has been shaped by the great history of philosophy - with the wholly new technical concept of self-similarity and self-affinity. Self-similar time series exist, for example, in dendrochronology, but also in music, in each case, across a certain scaling interval. The idea is to, again, turn this fact around, in order to apply it to the structure of the experienced time itself.

This new epoché by Ms Vrobel is non-trivial. It may be used to define a machine which, in a recursive way, generates an ever-richer Now. This "Now" machine would - paradoxically - be independent of any embedding into a certain time interval.

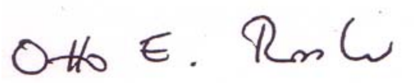
Ms Vrobel introduces here the novel concept of "condensation", which may be the most important technical concept of her work. There even arises - as she shows in the final paragraph of her work - an "ethical" problem. May we, at this stage, continue to think and build such a machine?

It is rare for works of philosophy to be directly convertible into a possibly dangerous technology. The mere possiblity of such a thought says something about the originality of the work in question.

We all know that scientific work is difficult and time-consuming, and that very few of the most original ideas survive. In spite of this, science lives off those few original ideas which emerge from such work. The number of original ideas in this work is far above average. The reader has undeniably the feeling of being witness to the emergence of a novel theoretical structure. One is impressed and mentally stretched by the emerging forming power. One is reminded of the originality of another Husserl disciple, Emmanuel Levinas.

Although I cannot presume to be able to pass categorical judgement on every aspect of this work, I am impressed by the technical mastery it exhibits. The essence of the theory of fractals is transferred, in a technically sound manner, into the sphere of the humanities. This alone is a lasting achievement. The fact that I feel convinced of having a significant work in front of me, is based not least on this technical aspect which is central to the work. The unpretentiousness of the author, who repeatedly stresses that only inital steps have been taken here, rounds off the picture.

I with great pleasure put this work into the hands of the reader.



Otto E. Rössler, Tübingen March 26, 1997

0. Introduction

There is, apart from temporal empirical knowledge (i.e. implying duration), a further, non-temporal access to cognition of temporal structures. A non-temporal access enables us to explain subjectively (in each case) varying empirical knowledge of duration, as well as *insight*[1] and precognition.

Access to cognition of temporal structures through temporal empirical knowledge works as an arranging of structures, in retrospect, of the relational temporal order through the *Now* of the modal temporal order[2]. Furthermore, this access renders possible the cognition of duration independent of the individual, i.e. duration in its limited form as a level-of-description-bound structure of incompatible states of facts. (Hereafter, the term "level of description" will be referred to as LOD.) This access does not provide, though, an explanation of subjectively (in each case) varying empirical knowledge of duration or a delineation of LOD-independent temporal structures.

A fractal concept of time differentiates the length, depth and density of time. If the length of time is determined by a LOD-generating subject, duration turns into a two-dimensional phenomenon: The length of time is generated through incompatible facts ("before-after-relations"), the depth of time through nested, compatible facts ("during-relations"). The density of time provides a means of measuring which is LOD-independent, in order to be able to compare different time series.

Non-temporal access to cognition of temporal structures becomes possible for self-similar nested structures: Self-similar structures provide, as congruent constants, the prerequisite(s) for time-condensation. The latter occurs in the case of the length of time approaching 0, the depth of time approaching ∞ and the extended present fitting, as the subject's position of empirical knowledge, congruently into the self-similar, nested structure. The non-temporal cognition of *primes*[3] by the subject, which is brought about by condensation, renders possible the cognition of the structure of these very primes on a different LOD. This kind of cognition generates an ad lib extendable present, since the

[1] A process of comprehension beyond the immediate present. The concept is introduced by Roger Penrose in his publication The Emperor's New Mind (Penrose 1989). In the present paper, an alternative definition of this concept of *insight* is worked out.
[2] The *Now* accords with Husserl's concept of an extended present.
[3] Nested structures of the B-series which do not exhibit nesting potential, and, therefore, cannot bring about further potential depth of time are, in the following, designated „primes“.

structure of the prime reaches, as seen from the (indexical) position of the subject, into the past as well as into the present. The process of cognition of a structure such as this does not involve duration, since condensation does not imply a succession in the form of incompatible facts, but is generated by congruent "during-relations".

If access to temporal structures were possible only through empirical knowledge, which implies duration, neither subjectively differing empirical knowledge of duration nor condensation through a deliniation independent of a LOD could be explained. This would lead to an alternative model to Penrose's concept of *insight*. A fractal concept of time provides both, and offers, beyond that, a model to explain precognition, since both past and future may be seized through nested primes.

1. Time and empirical knowledge of time

Is time real[4] or a mode of our empirical knowedge? Is time a function of an a priori scheme we impose on reality or is it possible to approach time through empirical knowledge? Starting with these questions, I shall try to show in the following chapter that, in order to avoid an infinite regress of prerequisites to possibilities, one must assume as (being) real a temporal structure which is independent of our empirical knowledge.

How could an approach towards the concept of time be brought about? Our access to the world is, at first, gained through empirical knowledge - therefore, a non-circular definition of time is not possible, for the defining individual is always already embedded in the subject matter he wishes to define: time.

Since we can only proceed from our own empirical knowledge of time, though, a potential access to a time which is independent of our empirical knowledge can only be obtained via that empirical knowledge. In order to achieve a differentiation of concepts, two delineations of time, neither of which can be reduced to the other, are investigated in the following chapter: the modal and the relational delineation of time. The modal delineation may be regarded as the time of the subject: it describes the flow of time from the past through the present into the future. The notion of time passing originates in this modal delineation.

Here, events appear to be, first, in the future, then present, and, finally, in the past. At the same time, our only direct access to the world, our only opportunity to act, lies in the present. Past and future are reflected in the present. This particularity of the modal delineation of time, the *Now*, has no counterpart in the relational delineation of time. Here, the concepts of *earlier*, *later* and *between* suffice to determine all permanent relations of events. The relational delineation of time is the time of physics, t, whose earlier-later relations are made comparable through the metrics of the mathematical continuum.

Neither of these two delineations can be reduced to the other: the *Now* has no counterpart in the relational delineation, and the special quality we attribute to the concepts of present and future - the present is remembered, the future anticipated - semantically surpasses the relational concepts of *earlier* and *later*.

In the following, the designations introduced by McTaggart[5], i.e. the *A-series* and the *B-series*, are employed to indicate the modal (A-series) and the relational (B-series) designations of time. Mc Taggart's proof of the unreality of time provides a convenient introduction to the question "In what relation do the A-series and the B-series stand to each other?", since that proof deals with and compares the properties of the modal and the relational delineations of time.

Mc Taggart's proof of the unreality of time states that

1. time essentially implies change,
2. change can only be explained by means of A-series-concepts,
3. notions of the A-series imply contradictions and can therefore not be employed for a description of reality, and, thus it follows that
4. time is unreal.

Point 3. requires elucidation. According to McTaggart, on the one hand, past, present and future are incompatible properties. On the other hand, every event is either past, present or future. If every event is past, present and future, every event must display incompatible

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[4] Here, *real* should be understood as *being independent of our empirical knowledge*.
[5] McTaggart 1908

properties. An event cannot display more than one of these properties, though. This is a contradiction.

McTaggart commits an indexical fallacy. Lowe[6] shows that the very words used by McTaggart to describe the problem are of an indexical nature:

> "'e is present' means, of course, 'e is happening now', and 'now' may usefully be compared with other indexical expressions like 'here' and ' I '. The truth conditions of utterances containing indexicals are context-dependent."[7]

The apparent contradiction implied in the A-series suffices for McTaggart to draw the conclusion that time is unreal, since he proceeded from the presupposition that time essentially implies change and change is an exclusive property of the A-series. Mc Taggart does not consider the B-series as real, since he regarded it as not sufficient for a construction of the concept of time.

1.1 On Husserl's attempt to reduce the concept of time to the A-series and the inevitability of assuming a real B-series

Husserl's attempt to ascribe empirical knowledge of time to the modal time order of the A-series was supposed to show that it was unnecessary to draw upon an objective time, i.e. the B-series, for an account of empirical knowledge of time. Husserl considers exclusively the subject as a time-generating element. His theory is based on the modes of empirical knowledge *retention*, *consciousness of the present*, and *protention*. The consciousness of the present represents, as the potential cumulation point of all retentions and protentions, past events by seeking it out in its (fixed) position and reflecting it, in a modified way, in the Now. A present such as this must exhibit extension, in order to be able to host both retention and protention. Exemplified by the perception of a series of notes as a tune, Husserl shows the necessity of assuming concepts such as retention and protention in order to understand our skill to recognize not only a series of isolated notes, but a tune[8]. He defines notes as so-called time objects (*Zeitobjekte*), which are, themselves, extended:

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[6] Lowe 1987
[7] Lowe 1987, p. 65
[8] Husserl 1928.

> "We refer to objects as *time objects in the special sense*, when they are not only units in time, but also contain temporal extension within themselves. When a musical note sounds, my objectivising apprehension may turn this musical note, which lasts and sounds, into an object. But it cannot do so with the duration of the musical note or the note in its duration. This note is, as such, a time object. The same is true for a tune, for any kind of change... Let us consider the example of a tune or an uninterrupted section of a tune. At first, this seems to be a simple matter: we hear the tune... While the first musical note sounds, the second comes, then the third, etc. Have we not to say: when the second note sounds, I hear it, but I no longer hear the first one anymore, etc? I do not, then, in truth, hear the tune, but only the individual present note. The fact that the section of the tune which has been played is objective to me, I owe - one is inclined to say - to recollection. And the fact that I do not, having reached the appropriate note, presume that that was all, I owe to anticipatory expectation... (the note) begins and stops, and its entire unity of duration, the unity of the entire process in which it starts and ends, 'shifts', after the ending, into an ever-more-remote past. In this receding motion, I still 'cling' to it, have it in a 'retention' and, as long as it lingers, it has its own temporality, it is the same, its duration is the same."[9]

In Husserl's phenomenology of the inner consciousness of time, time objects stand in a fixed relation to each other, and even recollection does not change this original order. Furthermore, Husserl claims that our consciousness not only perceives the time objects A and B (and, through the index of recollection, also A'and B'), but it also perceives succession:

> "This consciousness does indeed imply an A' and a B', but also a -'. Of course, this succession is not a third part, as if the way of writing the symbols consecutively indicated the succession. But still I can write down the following law:
>
> (A - B) = A' -' B'
>
> with the sense of: there exists a consciousness of the recollection of A and of B, but also a modified consciousness of 'A is succeeded by B'."[10]

Husserl is unable, though, to construct the succession of time objects without presupposing a B-series. Bieri shows[11] that Husserl's approach is contradictory, being based, on the one hand, on the timeless character of the subject but, on the other hand, describes reflexion in the consciousness of the present as a succession:

> "One will not be able to avoid interpreting this 'succession' as a real time structure. This is because it is phenomenologically inconceivable that a formally possible thought of a consciousness first constructs a succession and then places itself into that very succession and only in doing so manages a temporal presentation of its data."[12]

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9 Husserl 1928, p. 384ff (my translation).
10 Husserl 1928, p. 402 (my translation).
11 Bieri 1972 (my translation).
12 Bieri 1972, p. 197 (my translation).

Husserl's attempt to describe empirical knowledge of time by means of the A-series alone, wihtout falling back onto the B-series, fails, because his concept of reflexion already contains that of retention. If the A-series turns out not to be time-generating, a real B-series must be assumed.

The transcendental position too, which understands time as a prerequisite for rendering possible experience of any kind of reality, and therefore disputes the reality of time, turns out to be inconsistent, since

> "Kant's announcement that time is only a 'pure form of intuition' is unsatisfying not only because it does not sufficiently describe consciousness of time, but also because he does not, again, apply the transcendental question to this 'pure form of intuition'."[13]

It is inconceivable to regard empirical knowledge of time via the A-series as something subjectively generated, without assuming a futher level of generation, which (itself) again generates the time in which our consciousness of time works.

Thus, in order not to slip into an infinite regress, an account of empirical knowledge of time has to fall back onto a real temporal structure, which must be the B-series (and cannot, as pointed out above, be the A-series).

1.2 The relation between A- and B-series

The relation between A- and B-series cannot be exhaustively revealed in a simple mapping, which correlates events of the past to *earlier* and events of the future to *later*. Firstly, the *Now* of the A-series would have no counterpart in the B-series. Secondly, concepts correlating with the terms *past* and *future*, such as *memory* and *anticipation*, are not associated with the terms *earlier* or *later* as parts of the B-series.

Since, as was shown above, the A-series does not generate time, there are two conceivable relations between the A-series and the B-series:
1. Events of the B-series are interpreted through the A-series by the subject, via the *Now*, or
2. The B-series shows itself, in a modified way, in the A-series.[14]

Events of the B-series cannot be experienced directly. Our access to them has to occur via the subjective delineation of time, i.e. the A-series. If one does not want to deny the subject any kind of generating potential - this does not refer to a time-generating potential - one has to make relation no.1, which has events of the B-series interpreted via the *Now* of the A-series, the basis for all further considerations. Chapter 2.6. shows in what way the subject's generating potential may take effect via the A-series.

1.3 Duration

One significant characteristic of the relational delineation of time is the potential to compare and measure various events. This is possible because correlations with divisible units of the continuum can be established by means of a projection of events onto the mathematical continuum.

Such potential for measuring and dividing events rendered possible through delineations of the B-series does not necessarily make sense in the context of empirical knowledge of time, which can only be gained through the *Now* of the A-series. Bergson's concept of duration as a non-divisible whole dismisses ideas of juxtaposition and extension:

> "Let us therefore rather imagine the image of an infinitely small elastic band, contracted, if it were possible, into a mathematical point. We slowly start stretching it, so that the point turns into a line which grows continuously. Let us focus our attention not on the line qua line, but onto the action of pulling it. Notice that this action is indivisible, given that it would, were an interruption to be inserted, become two actions instead of one and that each of these actions is then the indivisible one in question. We can then say that it is not the moving action itself which is ever divisible, but the static line, which the action leaves under it as a trail in space."[15]

The Bergsonian concept of duration does not find its counterpart in the B-series, whose events, which are projected onto a/the mathematical continuum, are extended. Bergson's concept of duration is compatible with the modal delineation of time of the A-

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[13] Bieri 1972, p. 204 (my translation).
[14] This view has been developed by Bieri (Bieri 1972), who sees consciousness of time as a self-portrayal of real time.
[15] Bergson 1909, p. 8.

series, since it does not accept any juxtapositions or successions within the non-divisible whole of the duration (divisibility is a property of the B-series), but does contain past and future:

> "The internal duration is the continuous life of a recollection which extends the past into the present, so that the present may clearly contain the perpetually expanding image of the past.....Without this continuing existence of the past in the present, there would be no duration, only the existence of the moment."[16]

In the following chapters, the term duration is used in Husserl's sense, insofar as it implies the properties of an extended present which exhibits deep nesting of protentions and retentions. The predicate of Husserl's extended present is ascribed to events of the B-series which have the potential to form "during-relations". The terms *time condensation* and *prime*[17] in Chapter 3 are based on the Bergsonian concept of duration, which defines a present which implies the past *and* is, at the same time, indivisible.

2. Fractal Time

On account of the so foregoing considerations regarding the A- and B-series, this paper presupposes a real B-series which exists independent of our empirical knowledge. The subject is not assumed to be time-generating; it does have an impact on the structure of time, though. This thesis will be supported by a fractal concept of time.

A fractal concept of time will be developed in order to render possible a concept of duration which is independent of a level of description. In addition, a fractal concept of time allows a differentiated accord of the phenomenon of subjectively different durations of events which cover intervals of identical lengths in the B-series. Furthermore, a fractal concept of time provides an alternative view of Penrose's concept *insight*.

2.1 Arbitrary choice of the level of description

[16] Bergson 1909, p. 27f.
[17] At this point, it will suffice to understand the term *prime* as a structure which exhibits properties of both B-series elements and so-called V-series elements (i.e. the internal structures of the latter), which are non-temporal and non-modifiable. A definition of the term *prime* will be given under 3.4.

We can access structures of the B-series only via the *Now* of the A-series and we can recognise these structures only in retrospect. Consequently, an analysis of a phonogram[18] for example, can only be conducted in retrospect, when the actual sounds cannot be perceived anymore, i.e., when they already cover a position in the past within the A-series.

Such a retrospectively recognized structure can be displayed, for example, by means of a phonogram which registers various oral accounts of a story. The following example refers to a French text spoken by male and female German and French native speakers. The text was analysed in terms of the distribution of pauses during the speech act.[19] A methodological problem to be solved in this context was posed by the definition of *pause*. How long must a speech-free interval be to qualify for the designation *pause*? Apart from semantic characteristics, the length of the interval in question provides a significant criterion for the definition of a pause. The fixing of a minimum length for a pause, though, is subjective and arbitrary. Does 1/10th of a second suffice for a speech-free interval to be designated a pause or does it make more sense to consider only speech-free intervals of several seconds length as pauses?

A time series analysis juxtaposes various alternative units of the B-series. The height of the amplitudes in the printout is mapped against the units of the B-series. Intervals of 1/10th of a second, one second, three seconds, etc. are each designated as the yardstick unit ε. Speech-free intervals in the printout are subsequently measured in all of the units chosen: the first measuring comprises all speech-free intervals which last longer than 3 seconds, the second measuring considers only intervals which last at least 1 second and ignores intervals shorter than 1 second. The third measuring process registers all speech-free intervals which last longer than 1/10 of a second. Intervals shorter than 1/10 of a second are ignored and not registered as a pause, and so on.

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[18] A phonogram is a printout of amplitudes of varying volumes (of sound) mapped against the time of the B-series.

[19] Dechert & Raupach 1980.

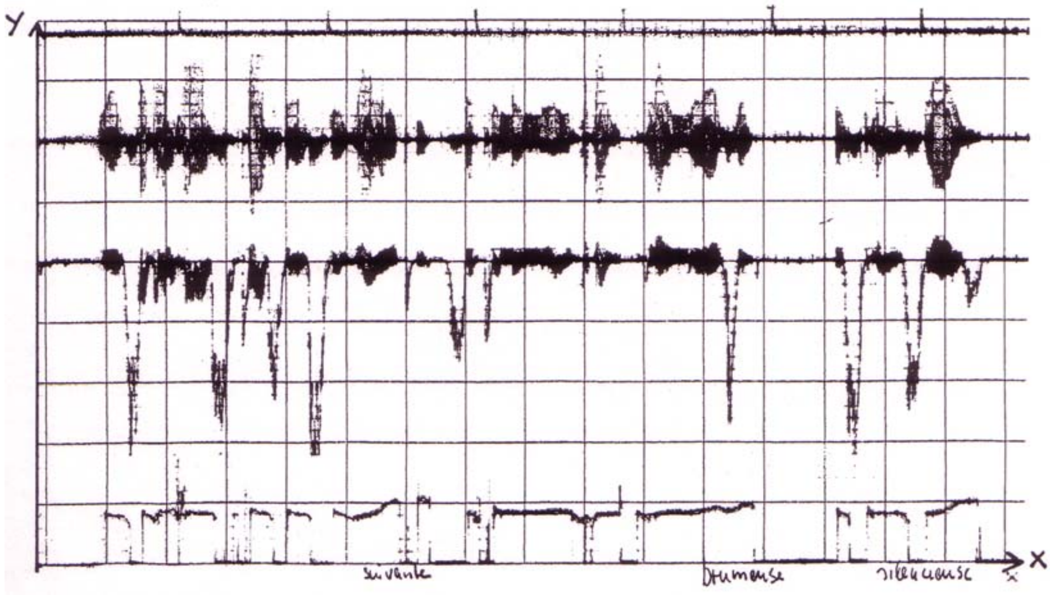

Figure 1

Native German speaker, male. Speech-free intervals are assigned values of y = 0.

This measuring cascade may be continued ad lib by chosing ever shorter intervals as pauses. If this is done, the choice of units which determine the measuring process (and, thereby, the measuring result) is made on a level of abstraction which highlights the criterion "length of a speech-free interval" among many other conceivable pause-defining characteristics (e.g. turning signals, etc.). On this level of abstraction, pauses are defined in terms of speech-free intervals of various lengths.

In the following, levels of abstraction such as these are referred to as levels of description. This term indicates that the investigation has a descriptive character[20]. Levels of description are defined subjectively and are, therefore, subject to a certain arbitrariness. Fractal time series analyses[21] provide one way of avoiding an arbitrary choice of a level of description and exhibit a more general picture of the distribution of pauses. At this point, a short excursion on the topic of fractals and self-similarity will be of use to facilitate an understanding of this method.

[20] According to Hofstadter, an explanation of a phenomenon is often a description of the same phenomenon on a different LOD: "Moreover, we will have to admit various types of ‘causality’: ways in which an event at one level of description can 'cause' events at other levels to happen. Sometimes event A will be said to 'cause' event B simply for the reason that the one is a translation, on another level of description, of the other." (Hofstadter 1980, p. 709) In this context, the terms explanation and description are, in the sense of Hofstadter, more closely related than they are elsewhere.

[21] See Appendix 4.2.

2.2 Fractals and self-similarity

A fractal is a structure which exhibits detail on various levels of description. [22] By modifying the yardstick by which a structure is measured (and, thereby, also the level of description, if this is defined by the yardstick used), e.g. by making it smaller, a measurement with a continually reduced yardstick will lead to the exhibition of ever more detailed structures.

To begin with, one must distinguish between spatial and temporal fractals, in order to avoid any misunderstanding regarding the concept of fractal time to be developed. We encounter spatial fractals in structures without characteristic size, i.e. they exhibit detail and can be described on different levels by looking at them through a magnifying glass or from a great distance. In contrast to temporal fractals, spatial fractals are perceivable through our (visual) senses and often draw attention to themselves through an observable particular internal organisation of their structure. If we look at a fern from a distance, we recognize the same structure we see looking at it at close quarters.

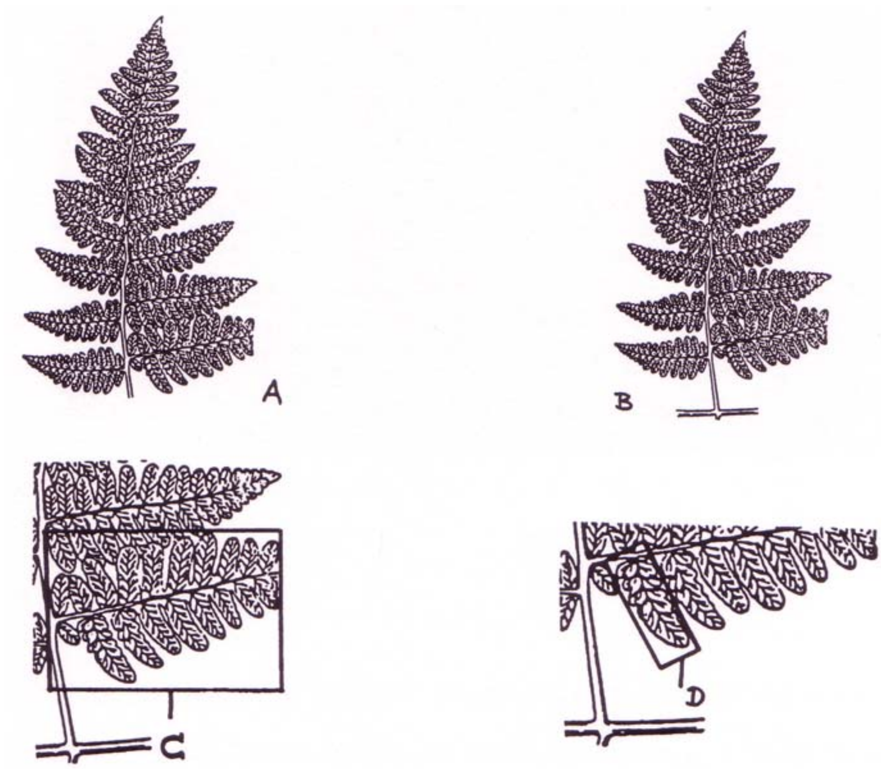

Figure 2

This invariance of a structure to a change of scale is called scale-invariance. Another term for this is self-similarity. While the latter term was coined by Mandelbrot,[23] the former cannot be conclusively attributed to any single source. Self-similar structures consist of

[22] A mathematical fractal exhibits detail on all levels of description within the fractal metric space H.
[23] Mandelbrot 1982.

copies of themselves. An additional effect is observable if one deals with a scale-invariant structure, such as that in Figure 3. If one looks at the photograph of the bottom of a swimming pool from which the water has been pumped out, it is next to impossible to determine the position
of the photographer.

This uncertainty is a result of the strong similarity of the dendroid structures, which look very similar on nearly all photographic enlargements and reductions. Without a point of reference with a characteristic size (e.g. leaves on the ground or the edge of the pool) it is impossible to make a statement about the size of these dendroid structures: they might be extremely large, such as a river delta (on a photograph taken by a satellite) or extremely small, such as rust particle on a steel surface (on a photograph taken with an electron microscope.

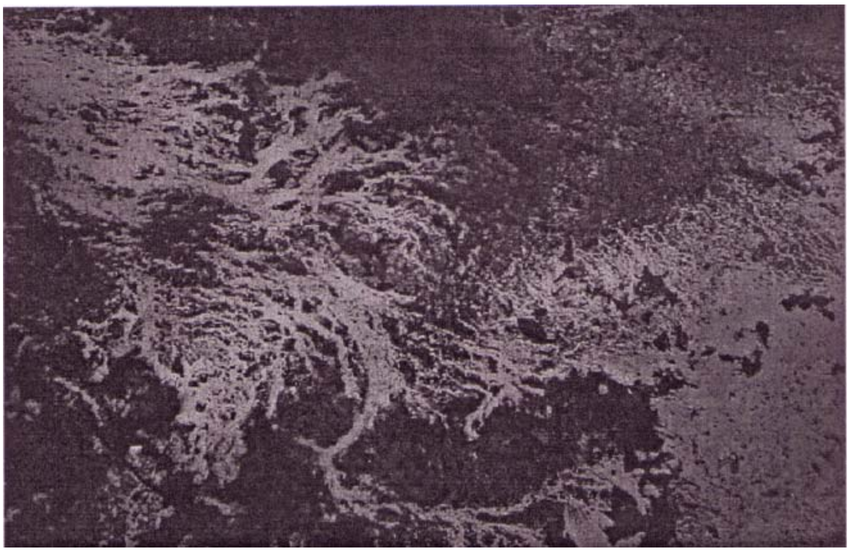

Figure 3

Self-similarity means invariance to a change of scale. A structure is self-similar, if it is symmetrical to a cjange of the level of description (where the level of desription is defined by the scale). For the purposes of my argument, the term symmetry is defined according to Weyl as an "invariance of a configuration of elements under a group of automorphic transformations."[24]

2.2.1 Fractal dimensions and statistical self-similarity

In order to be able to compare various fractal structures, the concept of the *fractal dimension* has been introduced. This concept has by no means been unequivocally defined. The relevant literature offers quite a zoo of definitions. All of these definitions aspire, by different methods and varying ranges of application, to capture a quantity which may innoculously be termed the *density* of a structure.

The first definition of the term fractal was given by Mandelbrot, [25] who elucidates the concept with the question *How long is the coast of Britain?*
The answer to that question varies with the yardstick used to measure the coastline. If the coastline is measured by means of the unit ε = 1 meter, tiny inlets which are too small to be measured by a yardstick of this size will be disregarded. Some of them will be taken into account if one measures the coastline again with a shorter yardstick of, say, ε = 10 cm. Adding up the results of the two measurements, one will find that the length of the coastline grows with every reduction of the yardstick ε: If a lizard walks the circumference of the island with steps of 3 cm length, the coastline will increase again, and an ant with a step length of just 2 mm will get an even larger result.

Could this go on ad infinitum? The mathematician Mandelbrot idealizes this example by assuming no limit to the reduction of the yardstick. This certainly makes sense in mathematics; in nature, though, such yardsticks and scale-invariant structures find their upper and lower limits. A continuing reduction of the yardstick ε will become meaningless in the subatomic realm, at the latest. There is always a point where the concept of *measurement* no longer makes sense.

A mathematical description allowing for an infinite number of gradations and, thereby, for an infinitely small ε, leads to the following problem: if several coastlines are infinitely long (measured by an infinitely small yardstick), then how can one compare them? Does four times infinity equal infinity? [26]

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[24] Weyl 1952, preface.
[25] Mandelbrot 1982.
[26] Cf. Mandelbrot 1982.

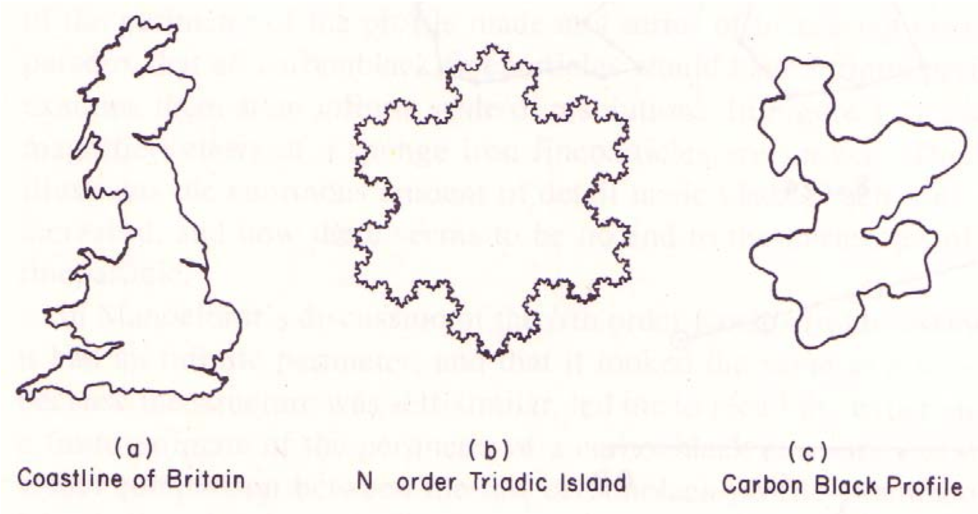
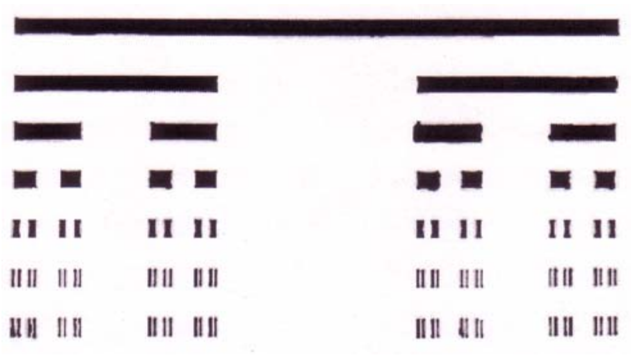


Figure 4[27]

In order to solve this problem, Mandelbrot suggests, as an alternative description, the use of the fractal dimension, which is determinable for self-similar structures. The fractal dimension is a quantity which may be determined independent of a level of description defined by a yardstick ε. Independent of subjectively arbitrarily chosen levels of description, it determines the density of a structure in a metric space.

According to Mandelbrot, the fractal dimension (d) of a self-similar structure may be determined by dividing the logarithm of the number of similar structures (n) by the logarithm of the scaling factor (s). The scaling factor is the factor by which the whole structure is reduced to a smaller version of the original.



For self-similar structures like the Cantor-dust, the fractal dimension is easily determined, since the regularity of the nesting pattern is captured at a glance.

Figure 5

The Cantor-dust is generated by reducing the nested structures by a third, from one level of description to the next, and mapping the reduced part onto the first and the third third of the initiator:

$$d = \frac{\log n}{\log s}\text{ , i.e., for the Cantor dust: }\frac{\log 2}{\log 3} = 0.6309....$$

This fractal dimension introduced by Mandelbrot [28] is often refered to as self-similarity dimension,[29] since this method permits the determination of a structure's density in a metric space only for self-similar structures. Since this self-similarity dimension can be visualized very clearly, it is a likely candidate for the introduction of the concept of a fractal dimension.

Figure 6[30]

The determination of the fractal dimension is based on mathematical models permitting an infinite nesting of ever-decreasing units. Cramer[31] points out that real objects, such as coastlines, deltas, ferns, etc., exhibit only a limited scale-invariance:

> "The concept of the fractal dimension and self-similarity is, to begin with, a mathematical one. For real physical and chemical objects, diffusion curves, surfaces of crystals or proteins, self-similarity will never be fully realized for all scales of length. There is an upper and a lower limit for it."[32]

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27 from: Kaye 1989, p. 1.
28 Mandelbrot 1982.
29 Grossmann 1988.
30 from: Stewart 1982, p. 21f.
31 Cramer 1988.
32 Cramer 1988, p. 172 (my translation).

The similarity present on different levels of description in natural structures, such as the dendroid structure in Figure 3, does not exhibit identical copies on each level of description, but only similar structures. Nevertheless, this scale-invariance, even though it may be imprecise, makes orientation impossible. One is not in position to determine the distance between the photographer and the pool, i.e., the observer and the object under observation. The strong similarity of natural structures observed at different LODs produces the same effect as for nestings of exact mathematical structures.

If the self-similarity dimension cannot be determined as a result of dealing with a mere similarity which is not based on exact copies, there is an alternative method of determining the fractal dimension of the structure in question: the so-called *Box Counting Method*. This method was developed by Barnsley [33] in order to determine the fractal dimension for both self-similar and non self-similar structures. The *Box Counting Method* determines a statistical self-similarity and, being applicable to natural as well as to mathematical structures, is a generalisation:

Theorem 1. *Let $A \in \mathscr{H}(\mathbb{X})$ where $(\mathbb{X}, d)$ is a metric space. Let $\epsilon_n = Cr^n$ for real numbers $0 < r < 1$ and $C > 0$, and integers $n = 1, 2, 3, \ldots$. If*

$$D = \lim_{n \to \infty} \left\{ \frac{\mathrm{Ln}(\mathscr{N}(A, \epsilon_n))}{\mathrm{Ln}(1/\epsilon_n)} \right\},$$

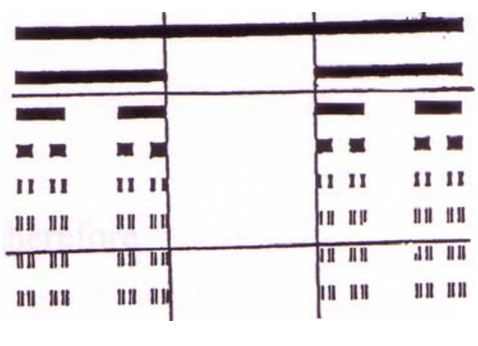
then A has fractal dimension D.

Barnsley's *Box Counting Theorem* [34]

For this reason of unlimited applicability, Barnsley's method will, in the following chapters, be used to determine the fractal dimension. In order that the reader may become acquainted with the *Box Counting Method*, it will be exemplified here by the determination of the fractal dimension for the Cantor-dust:

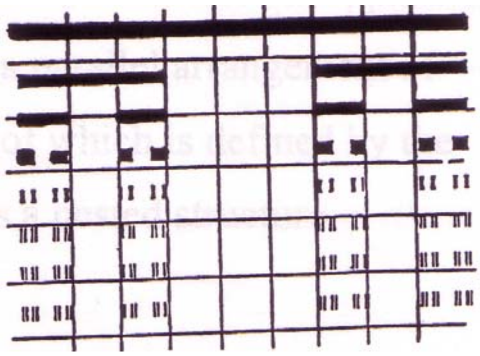
For squares of side length 1/3, n = 1,

ln 2 / ln 3



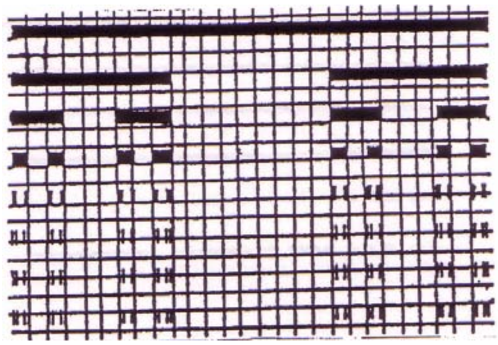
[33] Barnsley 1988.
[34] from: Barnsley 1988, p. 176f.

For squares of side length 1/9, n = 2,



$$\frac{\ln 4}{\ln 9} = \frac{\ln 2^2}{\ln 3^2} = \frac{2 \ln 2}{2 \ln 3} = \frac{\ln 2}{\ln 3}$$

For squares of side length 1/27, n = 3,



$$\frac{\ln 8}{\ln 27} = \frac{\ln 2^3}{\ln 3^3} = \frac{3 \ln 2}{3 \ln 3} = \frac{\ln 2}{\ln 3}$$

For squares of any side length, n = r,

$$\frac{\ln 2^r}{\ln 3^r} = \frac{r \ln 2}{r \ln 3} = \frac{\ln 2}{\ln 3}$$

The relation is independent of the scale used, therefore

$d = \ln 2 / \ln 3 = 0.6309 ....$

In contrast to spatial fractals [35] such as the one shown above, temporal fractals cannot be directly perceived. Possible self-similar structures may only be recognized in retrospect: structures of the B-series are only recognizable in retrospect through the *Now* of the A-series. Thus, pauses may be described in terms of fractal structures by arranging in parallel form speech-free intervals of various LODs, each level of which is defined by the appropriate scale. The result of this arrangement is a nested structure reminiscent of the Cantor dust:

[35] For further examples of the determination of fractal dimensions for mathematical structures of higher dimensions, such as the Koch curve and the Menger sponge, see Appendix 4.1.

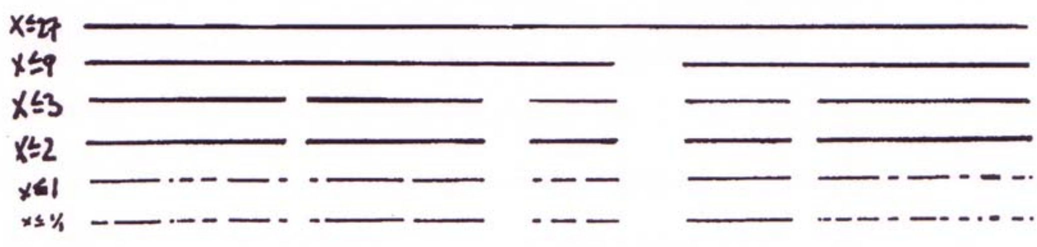


Figure 8

The number of pauses per LOD are added up and then related to their respective scaling factors. The fractal dimension may be determined, in analogy to the Cantor dust, by Mandelbrot's or by Barnsley's method, in order to obtain a quantity which is independent of scale. The result is essentially statistical, though: it does not imply any statement about the relations of individual pauses to each other. A so-called *Richardson plot* [36] provides a means for detecting self-similar structures. In order to detect self-similarity in the distribution of pauses, the number of pauses on each LOD are plotted, against the size of the pause-defining speech-free interval for each LOD, in/onto a log-log co-ordination system/scales. If the points are plotted on an imaginary straight line, such as the one in Figure 9, the structure is scale-invariant. The fractal dimension may be determined by Barnsley's Box Counting Method or simply by directly counting the individual pauses for each LOD.

Figure 9 shows the numbers of registered pauses, which are measured by an ever-decreasing grid of side-lengths 1/3, 1/9, 1/27, ...etc. The points representing these pauses are plotted as y-values, the side-lengths of the grids are represented as x-values in a log-log plot. For the grid size 1/3 (0.333...) of a previously fixed unit interval, 8 pauses covering this minimum length could be registered. Pauses covering shorter intervals are disregarded. The next LOD is defined by the scale 1/9 (0.111...) of the unit interval. It may be derived from the previous LOD by contracting the scale for considered pause intervals by another 1/3.

[36] The term *Richardson plot* is used in this context not as a plotting of the measured length of a perimeter, as defined by Kaye (1989): „A summary of data from a structured walk exploration of the perimeter of a rugged profile, plotted on log-log scales (...), is known as a *Richardson plot* (...).", but as a plotting of an interrupted line such as the one in the example in Figure 8. This interrupted line is measured in the same way as the perimeter for a *Richardson plot*. I regard this procedure as acceptable, since methodically, there is no difference in determining the fractal dimensions of interrupted and uninterrupted curves.

The generation of all further LODs runs analogously: in each case the scale is contracted from one LOD to the next by 1/3. Proceeding in this method, scale 1/3 registers 8 pauses, scale 1/9 registers 23 pauses, scale 1/27 registers 54 pauses, ...and so on. The quotient of the numbers of pauses and scaling factors equals a value approaching 1.1.... :

log 8 / log 3 = 1.8929...;
log 23 / log 9 = 1.4270...;
log 54 / log 27 = 1.2103...;
log 139 / log 81 = 1.1263...;
.....

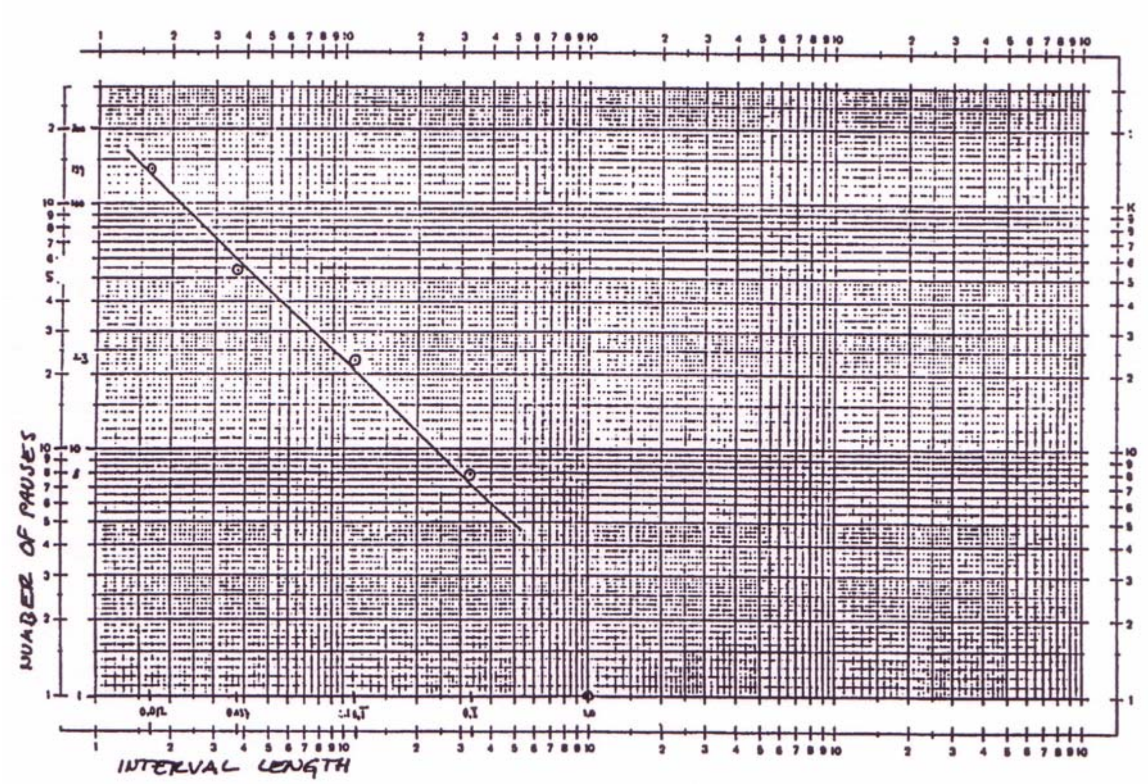


Figure 9

The Richardson plot shows the plotted points arranged on an imaginary straight line. This distribution reveals the presence of an, at least statistical scale-invariance. The number of registered pauses increases with every contraction of the grid scale but the ratio between the respective numbers of pauses and the scaling factors remains fairly constant (or, in some cases, approaches a limit point),[37] with the remaining abberations decreasing with every further contraction of the grid scale. The same method is used in Appendix 4 to determine the fractal dimension of the Koch curve. In contrast to the Koch curve, the nesting of pauses is not self-similar (other than statistically) and is bound by an upper and a lower limit. This method allows a LOD-independent delineation of all spatial and temporal structures.[38]

2.3 Fractal structures of the B-series: Δt_{length}, Δt_{depth} and $\Delta t_{density}$

In contrast to spatial fractal structures, temporal fractal structures are, apparently, not directly perceivable. The fractal and possibly self-similar structure of a B-series interval can only be determined in retrospect, e.g. by means of a time series analysis. The fractal dimension (d) determined above for pause distributions has, in contrast to other quantities, the following advantages: d is LOD-independent and therefore precludespossible arbitrary choices of LOD and d enriches the B-series, which measures the time of physics, t, with the topologically more complex concepts of the depth of time, Δt_{depth} and the density of time, $\Delta t_{length:}$

- The depth of time, $\Delta t_{depth,}$ is the number of nested intervals and, therefore, also the number of LODs considered;

- The length of time, Δt_{length}, is the number of incompatible intervals on one LOD. The units in which Δt_{length} is measured do not have to match those measured in the time of physics, t. A unit may be defined by the verse of a song, for example, where the verses cover different lengths of time intervals in t.
- The density of time, $\Delta t_{density,}$ is the fractal dimension, determined by the ratio of the number of incompatible intervals per LOD and the scaling factor, i.e. the factor which determines the contraction of the scale ε from one LOD to the next.

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[37] Appendix 4.1 shows this ratio approaching a limit point for a mathematically generated structure, exemplified by the Koch curve.

[38] The scale-invariance detected in the example plotted in Figure 9 happens to be an exception in the data I was given. In terms of pause distribution most cases exhibit a regular aberration for/under the scaling factor 3: For the LOD defined by grid scale 1/9 (i.e. a minimum length of 0.25 seconds), a very large number of pauses was registered for most cases. The number was too large to be assigned to the self-similar sector.
These differences in the extension of the self-similar sector/Bereich could not be correlated to the gender or nationality of the speakers, though. For our purposes, the pausology example is only to only to serve as a model for a LOD-independent method of analysis of temporal structures. A more rigorous analysis in terms of possible correlations exceeds the limits of this study.

The large number of publications on time series analysis in all conceivable fields of research which have appeared in recent years makes it hard to gain an overview on the subject. Here, it shall suffice to present the results of two studies exhibiting, just like the pausology example above, statistical self-similarity.

The scale-invariance in the rate of change of cotton prices discovered by Mandelbrot [39] suggests a correlation between daily fluctuations and long term changes, although they are attributed to very different causes. Short-term changes are attributed to random fluctuations, long-term changes to macroeconomic influences such as wars or recessions. As early as 1963, Mandelbrot discovered the scaling principle of price change:

> "When X(t) is a price, log X(t) has the property that its increment over an arbitrary time lag d, log X(t+d) - logX(t), has a distribution independent of d, except for a scaling factor."[40]

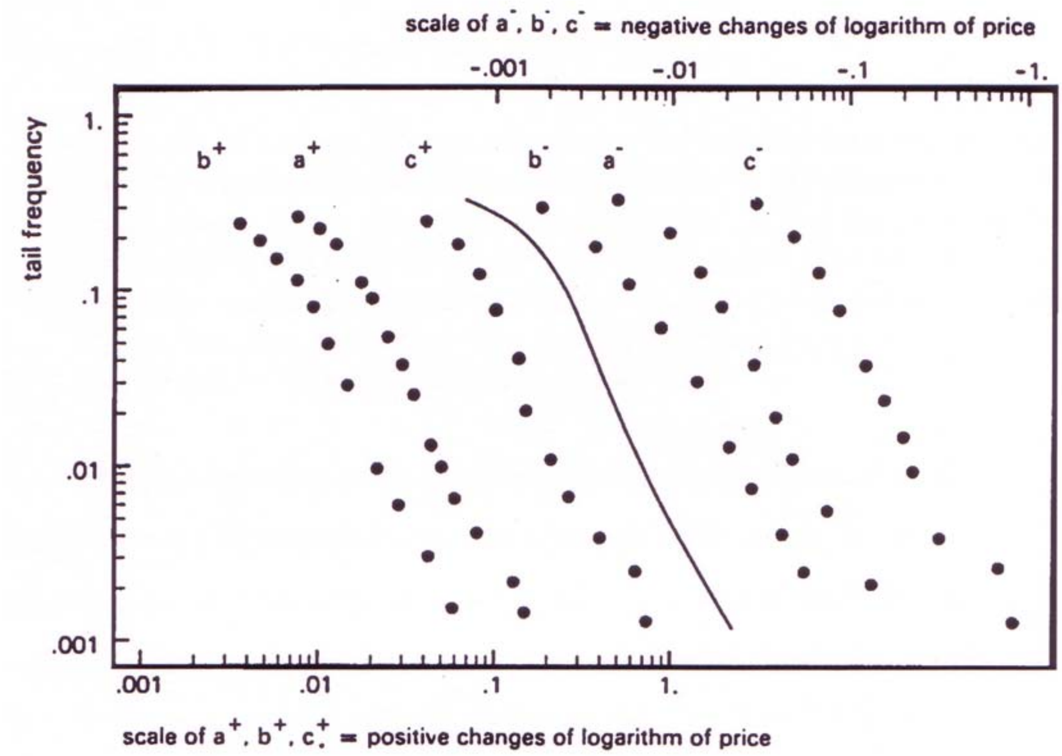

Figure 10 [41]

> "(a) X = $\log_e Z$ (t + 1 day) - $\log_e Z$ (t), where Z is the daily closing price at the New York Cotton Exchange, 1900 - 1905 (Data communicated by the U.S. Department of Agriculture).

[39] Mandelbrot 1982.
[40] Mandelbrot 1982, p. 337.
[41] from: Mandelbrot 1982, p. 340.

> (b) X = $\log_e$Z (t + 1 day) - $\log_e$Z (t), where Z is an index of daily closing prices of cotton on various Exchanges in the U.S., 1944 - 1958 (communicated by Hendrik S. Houthakker).
>
> (c) X = $\log_e$Z (t + 1 month) - $\log_e$Z (t), where Z is the closing price on the 15th of each month at the New York Cotton Exchange, 1880 - 1940 (communicated by the U.S. Department of Agriculture)."[42]

Kagan and Knopoff [43] concluded from their discovery of scale-invariance in seismic disturbances that it does not make sense to distiguish fore-, main- and aftershocks in such disturbances:

> "...almost all earthquakes are statistically and causally interdependent, a conclusion that contradicts attempts to divide the full catalog of earthquakes, either into sets of independent or main sequence events (aftershocks and foreshocks). If this picture applies even for the strongest earthquakes, and our result in the previous sections and elsewhere seem to confirm this, then all earthquakes occur in superclusters with very long time spans..."[44]

Recognition of scale-invariant structures may be regarded as an interpretation of this B-series structure through the A-series, i.e. as an achievement of the subject, or, according to Bieri, as a self-portrayal of the B-series.[45] Physical phenomena such as those described above, may be interpreted in both ways. In contrast to these, the examples presented in Chapter 2.6 can only be understood, if one presupposes a LOD-generating subject, which is able to influence and shape its time-perception.

The scale-invariance exhibited by the above-mentioned examples are of a temporal nature: processes, not spatial structures, are nested. The long-term behaviour of dynamical systems can be made visible by means of time series analyses. One transparent way of making it visible is the representation of the long-term behaviour of dynamical systems in phase space. Here, the patient observer will encounter a different king of self-similarity: chaotic attractors often exhibit self-similar structures in a phase space representation of all possible states of a system whose behaviour is governed by control parameters.

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42 Mandelbrot 1982, p. 340.
43 Kagan and Knopoff, 1981.
44 Kagan and Knopoff, 1981, p.2861.
45 Bieri 1972.

This kind of self-similarity is a virtual one, since the temporal development of a dynamical system does not necessarily correspond to a continuous curve of points in phase space. The point in phase space representing the next state of the system will be probably not be adjacent to the point representing the chronologically preceding state. After some strange loopings, though, spaces between points in phase space will probably be filled. The erratic jumping from one area of phase space to another will eventually produce a pattern, which may well be self-similar. The self-similarity inherent in the attractor presents the virtual self-similarity-blueprint of the system in question. A time series for Δt_{length} cannot reveal this kind of self-similarity - it is inherent in the set of all possible states the system may take.

This idea will not be extrapolated, since the concept of fractals suffices to deals with all the objectives in the Introduction to this paper. The role of chaotic attractors in the context of a fractal time model must be investigated elsewhere.

2.4 The Newtonian metric of time as a special case of fractal time metrics

By means of a thought experiment involving a fractal clock, I shall try to show in the following pages that the Newtonian metric of time may be regarded as a derivative of a fractal time metric. Such a fractal clock may be pictured as in Figure 11.

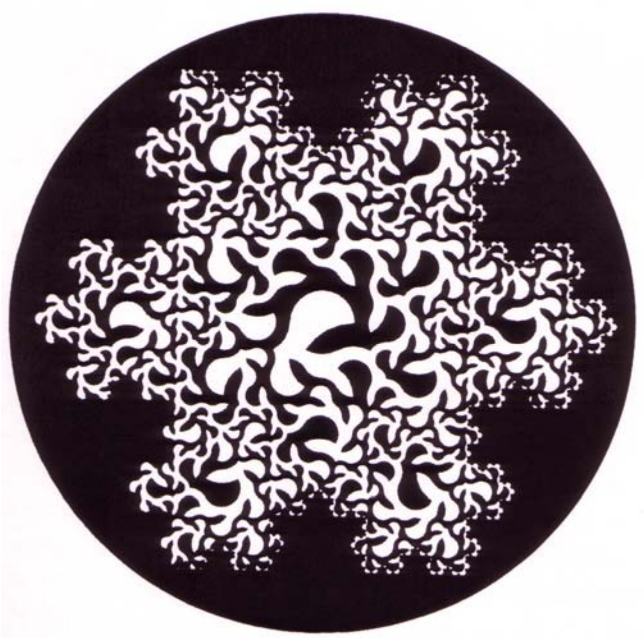

Figure 11[46]

Figure 11 shows a structure denoted as the *triadic Koch island* by Mandelbrot.

The generation of this structure is simple:

> "The construction begins with an 'initiator,' namely, a black Δ (equilateral triangle) with sides of unit length. Then one pastes upon the midthird of each side a Δ-shaped peninsula with sides of length 1/3. This second stage ends with a star hexagon, or Star of David. The same process of addition

[46] Mandelbrot 1982, p. 57.

of peninsulas is repeated with the Star's sides, and then again and again, ad infinitum." [47]

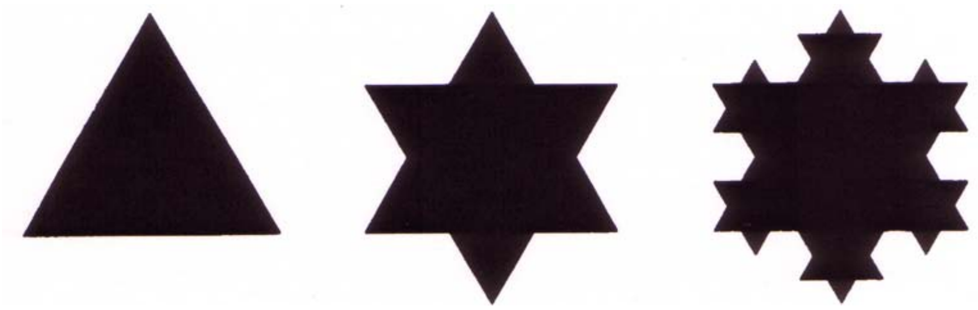

Figure 12[48]

Δt $_{depth}$: 3 ticks **during** 6 ticks **during** 18 ticks **during** ....

Imagine an infinite number of pointers attached to the perimeter of the triadic Koch island and with all pointers ticking away simultaneously, each at its own speed. You are imagining a fractal clock. This fractal clock ticks away just like any ordinary clock, except that there is an infinite number of pointers instead of just two (or three). The infinitely nested structure of the triadic Koch curve exhibits an infinite number of intervals, which the pointers of a fractal clock have to tick away. While pointer no. 1 ticks only three times (per lap), pointer no. 2 is ticking six times, pointer no. 3 is ticking eighteen times, and so on, ad infinitum.

Projected onto a one-dimensional straight line, the infinitely nested structure of the triadic Koch curve forms a continuum, and thereby, a Newtonian metric: the set of points generated in this way is the set of rational numbers.

Thus, the Newtonian metric may be defined in terms of fractals, as Δt_{length} of the nesting level ∞, i.e. $\Delta t_{depth} = \infty$.

2.5 Determination of Δt_{length} and Δt_{depth} without projection onto the mathematical continuum

In order to determine the quantities Δt_{length}, Δt_{depth} and $\Delta t_{density}$, LOD-defining units have to be, at least theoretically, projectable onto a mathematical continuum. It is possible,

[47] Mandelbrot 1982, p. 42.

though, to reveal the internal relations of individual LODs without such a projection. For the determination of Δt_{length} and Δt_{depth}, the internal relations between the individual LODs suffice.

These internal relations allow the determination of Δt_{length} and Δt_{depth} for cases in which a time series analysis turns out to be impossible, e.g. because the intervals of the B-series considered cannot be projected onto a mathematical continuum. I shall try to elucidate this point by determining Δt_{length} and Δt_{depth} for the song *Polythene Pam*. [49]

LODs for an analysis of this song may be defined by the following units: album, medley, song, verse, line, word and syllable. If the internal structure of the song is exhibited by arranging all LODs in a parallel way, a nested pattern is generated: The syllable *ly* is embeddded in the word *Polythene*, the word *Polythene* is embedded in the first verse, and so on. To allow an overview, here are all elements of the LOD defined by the unit *words*:

> WELL YOU SHOULD SEE POLYTHENE PAM SHE'S SO GOOD LOOKING BUT SHE LOOKS LIKE A MAN WELL YOU SHOULD SEE HER IN DRAG DRESSED IN HER POLYTHENE BAG YES YOU SHOULD SEE POLYTHENE PAM YEH YEH YEH GET A DOSE OF HER IN JACKBOOT AND KILT SHE'S KILLER DILLER WHEN SHE'S DRESSED TO THE HILT SHE'S THE KIND OF A GIRL THAT MAKES THE NEWS OF THE WORLD YES YOU COULD SAY SHE WAS ATTRACTIVELY BUILT YEH YEH YEH

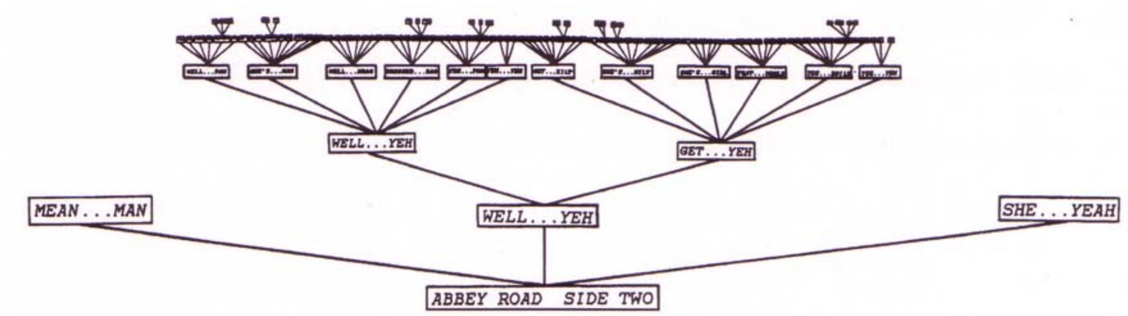

Figure 14

Above, the song is portrayed on five LODs: LOD syllable, LOD word, LOD line, LOD verse, LOD song. A parallel arrangement of these LODs generates 5 nestings.

The determination of Δt_{length} and Δt_{depth} does not require any reference to the measuring of the time of the B-series, which is structured by the mathematical continuum.

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[48] Mandelbrot 1982, p. 42.

The individual units are already defined by their mutual internal relations: the three syllables *Po*, *ly* and *thene* are congruent with the word *Polythene*.

The nestings in Figure 14 generate a depth of time $\Delta t_{depth} = 5$. This corresponds to the number of LODs or, respectively, the number of nestings. The length of time, Δt_{length}, is registered in the appropriate units:

Δt_{length} for LOD 1:	1 song;
Δt_{length} for LOD 2:	2 verses;
Δt_{length} for LOD 3:	12 lines;
Δt_{length} for LOD 4:	79 words;
Δt_{length} for LOD 5:	92 syllables.

Thus, the song *Polythene Pam* has a duration of 92 syllables, 79 words, 12 lines, 2 verses and 1 song. This data can be gathered independent of individual observation. The values for Δt_{length} and Δt_{length} are observer-independent, just as the values determined by time series analyses of physical measurements in the earlier examples. There is no subjective element contained in this method of determining the duration of a process, except for the choice of LODs.

2.6 Subjectively varying perceptions of duration

Subjective duration we experience or remember seems to evade all quantitive description. Processes of the B-series, which are shown by physical measurements to be of equal length, often appear to us to be of different length, be it during the moment of experiencing this process or in retrospect. This empirical knowledge of time is described impressively by Thomas Mann's character Hans Castorp [50]:

> "Emptiness and monotony may dilate the moment and the hour and make them 'tedious'; the great and greatest periods of time, though, they shorten and fade away even to nothingness. Conversely, rich and interesting content is capable of shortening and quickening the hour and even the actual day; on a large scale, though, it endows the course of time with breadth, weight and solidity, so that eventful years pass much more slowly than those poor, empty, light years which the wind blows before it, and which fly away. So, actually, what we call tedium is, rather, a pathological diversion

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49 Lennon/McCartney 1969.
50 Mann 1984.

> of time, resulting from monotony: in conditions of uninterrupted uniformity, great periods of time shrivel up in a manner which terrifies the heart to death....."[51]

Hans Castorp's experiences in Mann's *Zauberberg* become describable in terms of a fractal concept of time which distinguishes between the length and the depth of time. If one assumes a "rich and interesting content" to be synonymous with a B-series exhibiting many parallely arranged LODs and, therefore, a large number of nestings, then the depth of time increases with every newly added LOD. "On a large scale" may be regarded as being synonymous with recollecting a process which is, by means of the act of recollecting it, is embedded, in each case, in a larger interval, and, therefore, gains in depth. The past process is newly arranged, by means of recollecting, into a larger interval of the B-series including the present. This larger interval contains experiences had since that past process took place. By means of generating new nestings, i.e. new arrangements into larger intervals, these experiences relativise previous experiences which, in turn, have relativized the original process, as well as the original process itself. With the introduction of new LODs, the past process gains in depth. Thus, "breadth, weight and solidity" may be generated by the depth of time, which increases with every act of recollecting a past process, i.e. with every new nesting. (The alert reader may have noticed that the A-series has sneaked in again through the backdoor via the LOD concept as it it used in this context - a subjectively generated LOD. This is no accident. Though the subject may not be time-generating, it is, at least potentially, LOD-generating. This potential will be explained in Chapter 3.)

So much for the depth of time. The length of time is generated by arranging incompatible [52] processes which can be represented by B-series intervals. Arranging numerous intervals on just one LOD generates the momentary feeling of tediousness. This phenomenon may be explained as follows: The presence of only a few LODs is a result of the limited intake-capacity of our consciousness within a certain interval. Impressions which are perceived on only one LOD or a few LODs generate large extensions in Δt_{length}, through being incompatible. If these intervals are not nested, through recollection or reflection, into new, larger intervals, the result may be the fading of large periods of time as experienced by Hans Castorp:

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[51] Mann 1984, p. 110f (my translation).
[52] In this context, *incompatible* means *not-in-a-during-relation*.

"Emptiness and monotony may dilate the moment and the hour and make them 'tedious'; the great and greatest periods of time, though, they shorten and fade away even to nothingness."[53]

In this light, learning may be regarded as a new arrangement of shorter intervals within longer ones. Repeatedly new arranging of intervals in form of a continuous nesting plays an important role for our empirical knowledge of time: if a new experience is undergone, it modifies past experiences by means of further nesting. **New experiences can generate new LODs which, in turn, influence the subject's perception and empirical knowledge of time.**

Thus, the contrast of the never-ending summers of one's childhood to the seemingly ever-shrinking summers of adulthood may be attributed to the different numbers of LODs at hand and to those being generated. Although the child acquires a large number of new LODs through new experiencing and learning, the following situation will often ocurr: A child is trying to understand and share new experiences with his environment as well as possible by means of the metaphors already at his disposal. Often, the child prefers to use metaphors it is already familiar with and able to apply with confidence. In the following example, a small boy applies the spatial metaphor he is aready familiar with, rather than the temporal one which he has not mastered yet and which belongs to the adult world:

Question: "When did the boy jump the fence?"
Answer: "There!" (points to the fence (illustrated) in the book)

This example stems from H.H. Clark's essay "Time, Space, Semantics, and the Child"[54]. Clark attributes the behaviour just described to the acquisition of rules of application one has either already mastered, just as the boy in the example has already mastered the spatial metaphor, or one does not feel very confident about yet, just as the boy felt about the temporal metaphor most adults and older children would very likely have used.

Clark formulates the so-called *complexity hypothesis* which is based on the correlation between human levels of perception and the appropriate language levels[55]. The complexity hypothesis states that the order in which spatial concepts are acquired (to be exemplified here

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[53] Mann 1984, p. 110.
[54] Clark 1973.
[55] Clark 1973, p. 54.

by the acquisition of English prepositions) is imposed by (the acquisition of) rules of application which include direction, point of reference and dimension. If, of two terms A and B, B requires all the rules of application A requires plus an additional one, then A is acquired before B. This idea is illustrated for the prepositions *in*, *into*, and *out of*:

> *in* presupposes a three-dimensional space;
> *into* presupposes a three-dimensional space and a positive direction[56];
> *out of* presupposes a three-dimensional space, a positive direction and a negation of this direction.

According to Clark's complexity hypothesis, these prepositions are acquired in the following order: First *in*, then *into*, and finally *out of*. The complexity hypothesis makes the following further predictions:

(1) In antonymous pairs, the positive term will be acquired before the negative one (e.g. *into* before *out of*);
(2) *At*, *on* and *in* are acquired before *to*, *onto* and *into*, since the latter require, in addition, a direction;
(3) Location prepositions such as *at*, *on* and *in* are acquired before correlative location prepositions such as *above* and *in front of*, since the latter require, in addition, a point of reference;
(4) *Tall* and *short* will be acquired before *thick* and *thin*, since the latter require an additional dimension;
(5) Unmarked[57] terms will be acquired before marked terms. The positive term is acquired before the negative one and the positive term determines the dimension: *long* (+), *short* (-) ⇒ dimension: *length*.

The same is true for temporal terms based on spatial metaphors.[58] The acquisition of temporal terms presupposes the mastering of the correlative spatial term, i.e., spatial terms are acquired before temporal ones, e.g.: "John is walking *in front of* Mary" will be learned before

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[56] According to Clark, the term *positive* means, in this context, *in the stronger perceptual field.* Cf. Clark 1973.
[57] *Unmarked* means neutral, without connotation. *Marked* means not neutral, loaded with (usuallly negative) connotation.
[58] According to Clark, every temporal term can be traced back to a spatial term, and the latter, in turn, to a perceptual field.

"John will arrive *before* Mary". If the child has not yet mastered the temporal rule of application, he will answer by using spatial rules of application: Question: "When did the boy jump the fence?"; Answer: "There." or "Right there."

The term "rule of application" corresponds to the term LOD (level of description) introduced in Chapter 2.1. In the light of a fractal time concept, learning is a result of generating new LODs. These LODs may be nested again and again by the subject through new experiences and recollecting. This will lead to a further increase in Δt_{depth}. Δt_{length} is contracted through the generation of "during-relations", since compatible events do not have to be arranged in sequence on a single LOD of the B-series.

The boy who answered a question concerning the temporal relations of events by means of spatial relations, since he had not yet mastered the rules of application for temporal relations, arranged all temporal relations, together with the spatial ones, on one LOD he has already internalized. The events are incompatible and therefore dilate Δt_{length} considerably. This is not the only example of an over-generalization leading to a dilated Δt_{length}. Adults and older children, who have generated numerous LODs, are in a position to arrange events in nestings, i.e. in "during-relations", and thereby dilate Δt_{depth}. This differentiation could explain why a summer of one's childhood is so incomparably much longer in comparison to a summer of one's adulthood (assuming the adult has generated more LODs than the child).

New rules of application or, alternatively, new LODs, can be generated through learning or recollecting (which may be denoted as learning too, since the act of nesting is that of arranging past events in a new context). New nestings often occur in clusters, i.e. in situations in which past facts are rearranged by innumerable recollection performances. Class reunions, housewarming parties, slide-shows on Christmas Eve, and the like serve as good examples for such recollection clusters. During such events, recollected facts are often nested over and over again, anf thereby newly arranged, as old stories are discussed, corrected and retold by individuals.

Through recollecting and newly arranging past facts on new LODs, Δt_{depth} increases perpetually. Δt_{length}, in contrast, seems to contract. During a class reunion, time seems to fly (unless the pityable families of the former class members were invited too. For them, Δt_{length} increases steadily, since they are not able to join in the recollecting and have to arrange everything they experience on a constant number of LODs - in other words, they are bored stiff).

This familiar phenomenon of subjectively varying perception of duration can be described by means of a fractal concept of time, i.e. by distinguishing Δt_{depth} and Δt_{length} of the B-series as well as assuming a LOD-generating subject. This provides a more differentiated view than the one suggested by Bieri[59], which assumes the self-portrayal of real time to equal consciousness of time.

The direction of the macroscopic arrow of time cannot be directly deduced from the fractal concept of time introduced in this paper. A delineation of the direction of time has to consider, though, the differentiation of Δt_{depth} and Δt_{length}, since Δt_{length} can, as an arrangement of incompatible facts on one LOD, only be conceived of after the determination of this LOD in Δt_{depth}. Δt_{depth} logically precedes Δt_{length}.

A direction of time presupposes a temporal arrangement. Any temporal arrangement is "held together" by "during-relations" which, by defining a LOD, rule out other LODs, which then provide a frame time, a reference for all levels of Δt_{depth}. Therefore, an arrow of time presupposes Δt_{depth} and Δt_{length} - Δt_{length} alone cannot specify an order without reference to a framework provided by at least one other LOD. Without the assumption of Δt_{depth}, there is no basis for the existence of an arrangement of incompatible facts. This existence is, in turn, the presupposition for the potential existence of a direction.

Apparently, a fractal concept of time cannot resolve the contradiction inherent in the relation of micro-reversibility and macro-irreversibility[60]. It reveals, though, that a single LOD is not sufficient to explain the arrow of time - Δt_{depth} is a necessary component.

3. Condensation

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[59] Bieri 1972.

[60] The direction of time inherent in a fractal arrow of time might be deduced after all. Cf. Vrobel, Susie, "Ice Cubes And Hot Water Bottles" in: *Fractals.* An Interdisciplinary Journal on the Complex Geometry of Nature. **Vol. 5** No. 1, 1997 World Scientific, Singapore, pp. 145ff.

Before venturing a fractal description of the concept *insight* as introduced by Penrose[61], it is necessary to define the concept of *condensation*, in order that the ideas presented in this chapter might be fully appreciated.

Condensation is a property generated by congruent nestings. It can be measured in the quantities of condensation velocity v(c) and condensation acceleration a(c). The basic quantities for the determination of v(c) and v(a) are Δt_{depth} and Δt_{length}. The quotient of Δt_{length} of LOD 1 and Δt_{length} of LOD 2 equals the condensation velocity v(c) for LOD 2 ⊂ LOD 1[62] (provided the units of both LODs can be converted to one another). For scale-invariant structures, v(c) is identical with the scaling factor s.

The quantities introduced above will be illustrated by means of their application in three examples. Examples 1 and 2 (Figures 15, 16) show the determination of the condensation velocity for scale-invariant fractals, here for the Koch curve and the Cantor dust. For all scale-invariant mathematical fractals, the condensation velocity is identical to the scaling factor s. Since it is constant for all LODs, the condensation acceleration for scale-invariant mathematical fractals equals 1. The fractal structure in Example 3 (Figures 17, 18) is based on dendrochronological data and is, as a natural fractal, bound by an upper and a lower limit to its scale-invariance. Here, the condensation velocity and acceleration have to be determined separately for each individual relation between two neighbouring LODs. Dendrochronological data provides excellent material for time series analyses, since large amounts of data have been gathered over very long time spans: the change of the width of growth rings of oak trees in Europe was published by Fletcher[63].

In the examples from the field of dendrochronology presented below, the unit *year* was chosen for Δt_{length}, since a growth ring corresponds exactly to the growth of a treetrunk during one year. The condensation acceleration equals the quotient of two condensation velocities.

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61 Penrose 1989.

62 ⊂ denotes *nested in*.

63 Fletcher 1978.

Example 1:

$$v(c) = \frac{\frac{4}{3}}{\frac{4}{9}} = 3$$

$$v(c) = \frac{\frac{4}{9}}{\frac{4}{27}} = 3$$

$$v(c) = \frac{\frac{4}{27}}{\frac{4}{81}} = 3$$

$$v(c) = \frac{\frac{4}{81}}{\frac{4}{243}} = 3$$

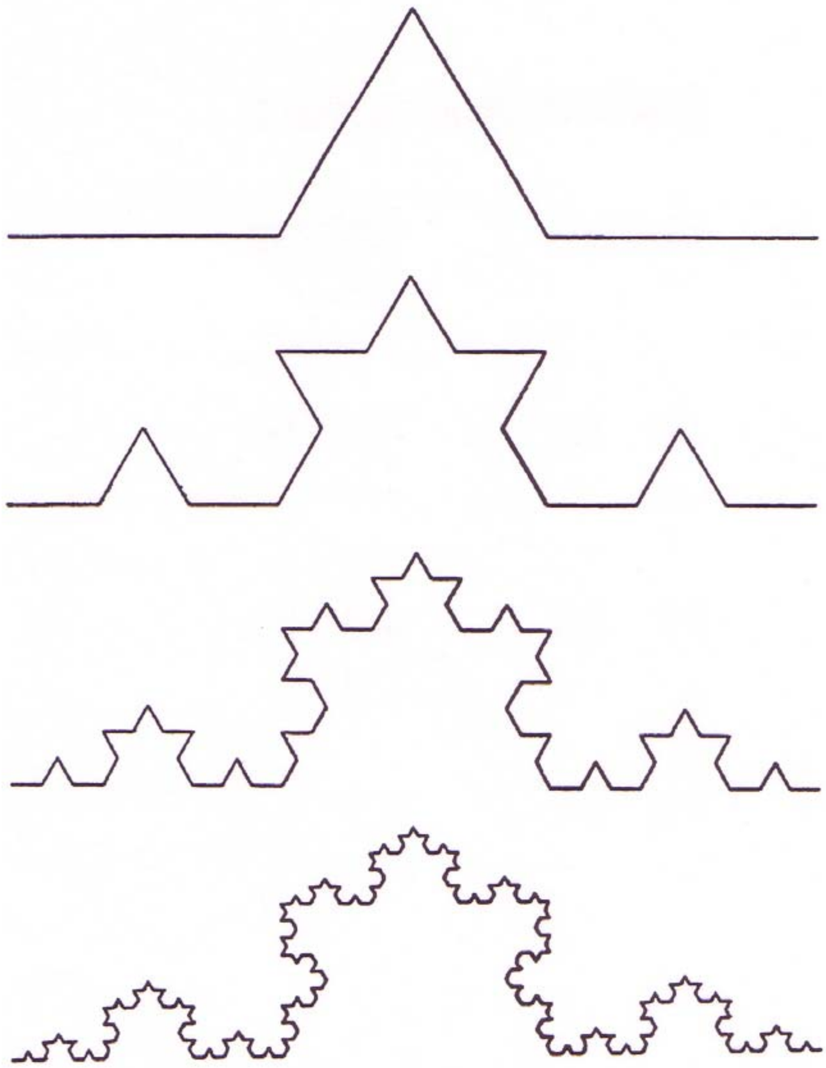

Figure 15

The condensation velocity v(c) = 3 is constant for the Koch curve; the condensation acceleration a(c) = 1.

In contrast to the Koch curve, whose length increases with every iteration, the extension of the Cantor dust decreases step by step with every iteration. Here, too, the condensation velocity v(c) is constant (v(c) = 3); the condensation acceleration also equals 1.

Example 2:

$$v(c) = \frac{136}{45.3} = 3$$

$$v(c) = \frac{45.3}{15.1} = 3$$

$$v(c) = \frac{15.1}{5.037} = 3$$

$$v(c) = \frac{5.037}{1.679...} = 3$$

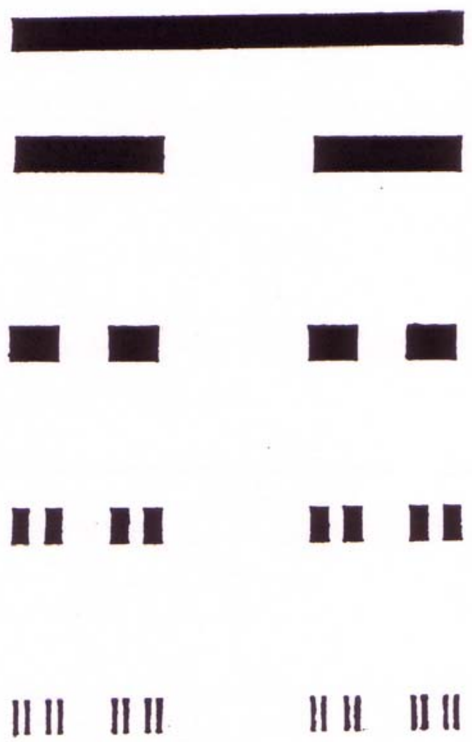

Figure 16

v(c) is constant,

a(c) = 1.

Fletcher's data on growth rings[64] was scrutinised by the present author for scale-invariance by means of arranging the results of measurements with various ε in the following manner: LOD A registered all rates of change in the width of growth rings with a delay of 17 years; for LOD B, the intervals between measuring steps were reduced to 7 years; LOD C measured the rate of change in tree widths annually. Several strings of data gathered on various LODs exhibit scale-invariant sequences, such as the following examples from the interval 1800-1960.

Example 3.1:
The scale-invariant pattern which appears on LOD A and LOD B (a) and the corresponding time scales (b).

(a) (b)

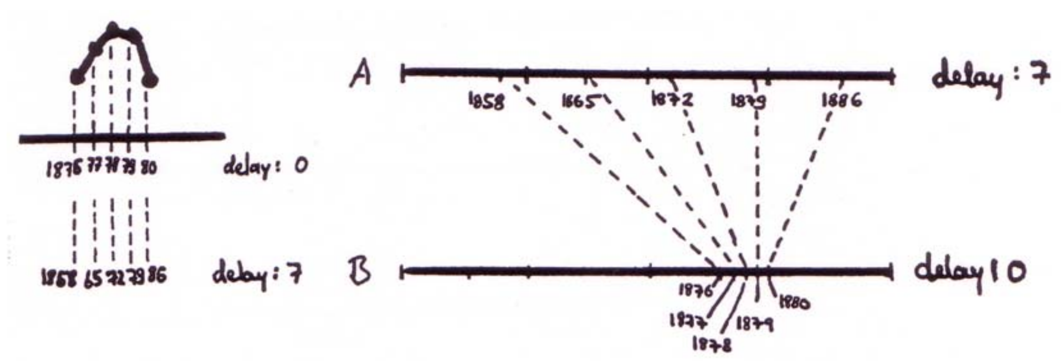

Figure 17

Example 3.2:
The scale-invariant pattern which appears on three LODs (a) and the corresponding time scales (b).

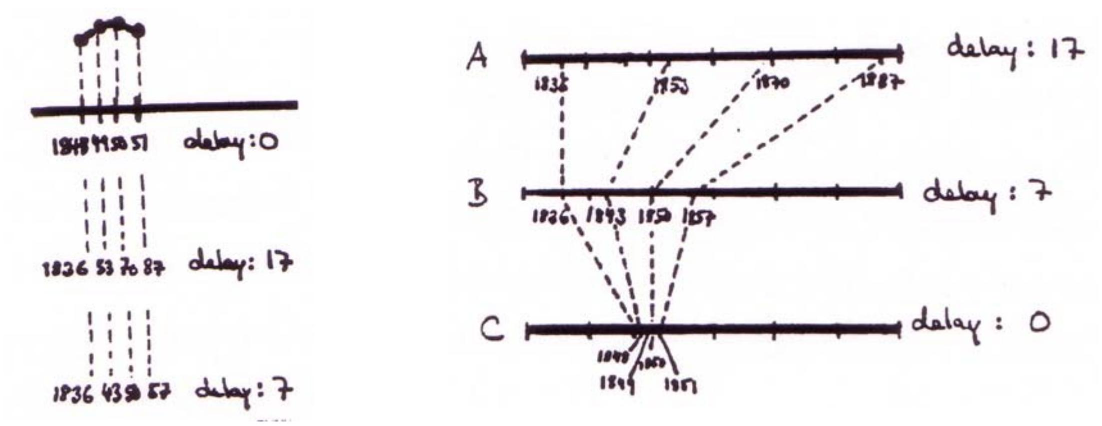

Figure 18(a) (b)

If long and short time intervals of the B-series exhibit the same pattern of change, we may define this shared pattern as a constant which serves as a reference scale in order to arrange the (internal) relations of various LODs. This relation may be illustrated through the individual condensation velocities and accelerations. What purpose all this may serve will be dealt with in the next chapter.

3.1 Roger Penrose's concept *insight*

[64] Fletcher 1978.

In his book *The Emperor's New Mind*[65], Roger Penrose seeks a new quantum theory CQG (Correct Quantum Gravity) which is intended to bridge the quantum world and the classical world, including general relativity, and describe how non-algorithmic elements of the quantum level are catapulted up to the macroscopic level of our consciousness. This new quantum theory does not exist yet - Penrose only shows that, in the so-called R-part[66] of quantum theory, a non-algorithmic element may be found, which is to be catapulted to the macroscopic level (in order to deduce the arrow of time present on the macroscopic level). According to Penrose, non-algorithmic elements are also non-temporal elements, since there is involved no computation, which is bound to take place in time, i.e. covers an interval of the B-series.

Penrose hopes to be able to develop a *physics of the mind*, in which human consciousness provides the pivot between the physical, time-asymmetrical world of algorithms and Plato's timeless world of ideas. Contact with Plato's world of mathematical ideas occurs, according to Penrose, in a non-temporal manner, i.e. no time passes "during" this contact. This process, which he calls *insight*, is non-algorithmic. The connection of our consciousness to the "real" physical world of algorithms, in which time must pass whenever information is transmitted, is time-asymmetrical. Penrose illustrates this idea of distinguishing non-temporal and time-asymmetrical worlds through an example in which he describes the experience of *insight*:

> "An extreme example (...) is Mozart's ability to 'seize as a glance' (*sic*) an entire musical composition 'though it may be long'. One must assume, from Mozart's description, that this 'glance' contains the essentials of the entire composition, yet that the actual external time-span, in ordinary physical terms, of this conscious act of perception, could be in no way comparable with the time that the composition would take to perform."[67]

Penrose attributes such time-skipping vision also to the composer Bach. The experience described below can only be undergone if, on the one hand, the composer has

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65 Penrose 1989.

66 According to Penrose, the R-part of quantum theory (the part corresponding to the wave function collapse, i.e. for those cases in which a measurement takes place) is time-asymmetrical, in contrast to the U-part (the part which may be described by the Schrödinger-equation, i.e. for those cases in which no measurement takes place). The arrow of time does appear if one describes the quantum world from a classical LOD, i.e. if one carries out a measurement.

67 Penrose 1989, p. 575.

organized his tune in such a way that the character of the entire composition may be anticipated in even the tiniest elements and, on the other hand, the listener is experienced enough, i.e. has acquired enough LODs to be able to perceive and anticipate these structures.

> "Listen to the quadruple fugue in the final part of J.S. Bach's Art of Fugue. No-one with a feeling for Bach's music can help being moved as the composition stops after ten minutes of performance, just after the third theme enters. The composition as a whole still seems somehow to be 'there', but now it has faded from us in an instant. Bach died before he was able to complete the work, and his musical score simply stops at that point, with no written indication as to how he intended to continue. Yet it starts with such an assurance and total mastery that one cannot imagine that Bach did not hold the essentials of the entire composition in his head at the time. (...) Like Mozart, he must have been able to conceive the work in its entirety, with the intricate complication and artistry that fugal writing demands, all conjured up together. Yet, the temporal quality of such music is one of its essential ingredients. How is it that music can remain music if it is not being performed in 'real time'?"[68]

According to Penrose, he is himself familiar with this kind of flash-like insight, which apparently occurs in a non-algorithmic form. He describes how he was suddenly struck, while crossing a street and in the midst of a completely different chain of thought, by the solution to a physical problem (namely, the *point of no return* during the collapse of black holes). The time-span necessary to perform a reflection of this solution in no way corresponded to the temporal extension of the insight, which, according to Penrose, as it was of a non-algorithmic nature, was also of a non-temporal character.

Such clustered insights, which occur in a flash, i.e. which are received by the subject without (in an idealized way) any temporal extension being involved, are not limited to highly gifted recipients:

> "Even the impressions of memories of [an individual's] own time-consuming experiences seem somehow to be so 'compressed' that one can virtually 're-live' them in an instant of recollection."[69]

Musical examples of insight provide excellent candidates for fractal descriptions, since numerous studies on the topic of scale-invariance in music are already available.[70]

3.2 A case differentiation for a fractal description of the process *insight*

[68] Penrose 1989, p. 576.
[69] Penrose 1989, p. 576.

Fractal structures of the B-series can only be determined in retrospect, via the Now of the A-series. Through this ever-changing Now, we recognize (in retrospect, by means of an analysis of B-series data) scale-invariant structures as well as structures which exhibit no scale-invariance. The large amount of fractal structures we can observe should make us suspicious - is it the world or our way of thinking that is fractal? These considerations lead to the following questions:

(i) Does a scale-invariance present in the B-series reveal something about the very nature of this B-series, and, possibly something about a V-series existing beyond the B-series?

(ii) If we assume that it is possible to reveal a scale-invariance for all structures - what time-structuring possibilities result from this?

A case differentiation is required here, since the fractal model of time chosen as the basis for an alternative description of Penrose's concept *insight* heavily depends on the presupposed relation of the A- and B-series.

Case 1. Let us suppose the events of the B-series to present themselves in a modified way in the A-series. This is Bieri‘s position: he interprets consciousness of time as the self-portrayal of real time. No retentional nesting is possible, since the subject does not, in this case, have any LOD-generating potential, and can therefore not perform any nestings of structures of various LODs. Without nested structures, no scale-invariant structures can be recognized, and no fractal description of *insight* through time condensation is possible. It is possible, though, to carry out investigations on scale-invariance that are independent of the individual: scale-invariant structures of the B-series, which "portray themselves" can be determined for data such as is gathered from dendrochronological investigations, as in Figure 19.

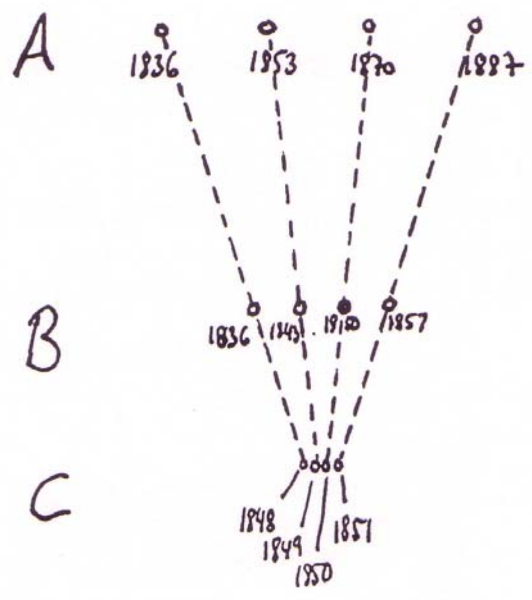

Figure 19

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[70] Cf. in particular: Voss 1988 and Hsü 1993.

Case 2. Let us suppose that our consciousness is in a position to determine scale-invariant structures present in the B-series via the Now of the A-series and that our consciousness is LOD-generating through the A-series. Then intervals, into which past facts are embedded by means of recollection are nested deeper and deeper with every act of recollection.

In this paper, Case 2 is assumed, since Case 1 can neither describe the phenomenon of subjective duration nor allow a fractal description of Penrose's concept *insight*. Case 2 attributes a creative role to the subject as the LOD-generating Me: time-perception becomes manipulable. Through the acquisition of LODs, Δt_{depth} becomes nearly arbitrarily extendable for everyone - Hans Castorp's light years, which the wind blows before it, are avoidable. Furthermore, the recognition of scale-invariant structures possibly allows a glimpse into an arbitrarily extendable present.

3.3 Assumption of a non-temporal V-series

Suppose the basic structure of a fractal consists of the sequence of musical notes f, a, c, f, or, alternatively, e, d, c. In Fidure 20, several nestings of this structure can be found:

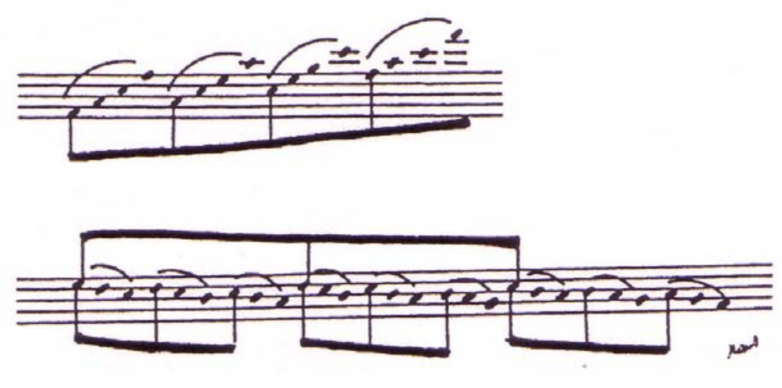

Figure 1: Several nested layers

Figure 20[71]

> "Musical events can be understood as occurring in numerous simultaneous layers, some brief and some lengthy. Self-similarity occurs between macroscopic patterns and the shorter patterns that comprise them. The simplest examples are created by a technique called melodic sequence, where a short sequence of notes with a particular pitch contour is several times repeated to create a longer sequence of short sequences. Each repetition of the basic sequence is displaced to a new pitch level; the contour of

[71] Mayer-Kress et al 1993, p. 13.

the macro-sequence is created by a pattern of displacements that replicates the contour of the basic sequence. Replications may be nested several layers deep..."[72]

The correlation of long and short structures in a nested scale-invariant composition allows the listener to anticipate the character of the entire composition when listening to only a short section of the composition. If this short section is from the middle of the entire composition, it is possible to catch a glimpse, audially speaking, of parts of the composition already played as well as of those yet-to-be played. In terms of the A-series, one learns about past and future structures via an extended but indivisible present which accommodates the basic structure of the fractal it is embedded in.

Scale-invariances in compositions are bound by an upper and a lower limit to the nestings. This is also true for most natural fractals[73]. Picture an infinitely nested temporal fractal, in the shape of the Koch curve, for example, and you will find the basic structure of the fractal on all LODs. This structure, which is present in all nestings, measures a different interval Δt_{length} for each LOD, i.e. for each B-series on all levels of nesting.

The shape of the Koch-curve (Figure 21) has been chosen for our purposes, since it is illustrative and allows the observer to seize at a glance the scale-invariance implied. The Koch curve does not correspond to any "real" physical process.

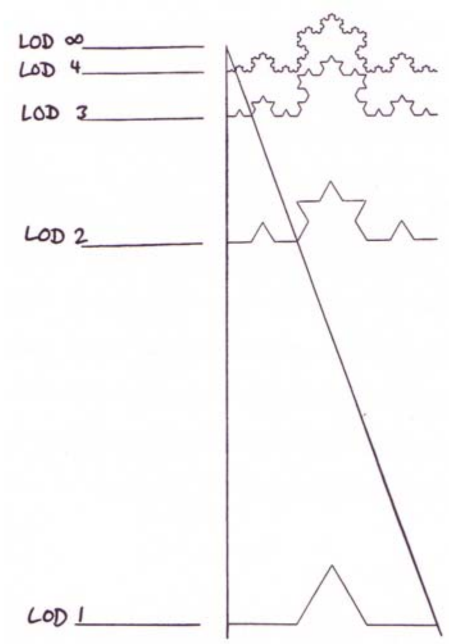

Figure 21

The structure of a fractal may be defined by the appropriate B-series interval it covers. In the case of the Koch curve, the structure corresponds to varying intervals of Δt_{length}, depending on the LOD in question. In order to give a LOD-independent definition of the structure, i.e. neither in terms of its length, nor in terms of its earlier-later relations, concepts are required which allow a description of the

[72] Mayer-Kress et al 1993, pp. 12/13.

structure without implying the idea of an extension. Such concepts can be found in Plato's world of ideas: timelessness, unextendedness, indivisibility. At first sight, McTaggart's C-series, too, could accommodate the structure of fractals, independent of their extensions in Δt_{length}. Mc Taggart defines his C-series as

> "a series of the permanent relations to one another of those realities which in time are events - and it is the combination of this series with the A determinations which gives time. But this other series - let us call it the C-series - is not temporal, for it involves no change, but only an order."[74]

The C-series implies an element ("not temporal") of Plato's world of ideas, but only marginally qualifies for the description of fractal structures. McTaggart's C-series does not correspond to Plato's world of ideas, since for McTaggart, time only exists as a combination of the A-series with the C-series, and the B-series comes into existence only through this prior combination. Since this paper presupposes the existence of a B-series independent of any observer's position (cf. Chapter 1), McTaggart's C-series cannot be used to describe fractal time.

In order to demarcate the concept required to describe the phenomenon *insight* against the background of existing concepts of non-temporal orders, the following definitions are needed:

Definition of V-series. The V-series is the set of structures which are non-temporal (i.e. not implying any duration), unmodifiable and inaccessible. The structures of the V-series exist in the (duration-implying) B-series as structured intervals of Δt_{length}.

The need to assume a non-temporal (i.e. implying no duration) V-series results from the lack of a sector to which that last nested structure of a fractal can be appropriately attributed. (This last nested structure corresponds to the upper limit of the fractal.)

Elements of this sector (which accommodates the last nested structure of a fractal) must exhibit some characteristics of both the B-series and the V-series. In order to define such an interim-sector (between B- and V-series), one must assume the existence of a V-series,

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[73] Cf. Cramer 1988.
[74] McTaggart 1908, pp. 461/462.

since some of the characteristics of V-series elements define this interim-sector. Let me describe the elements of this interim-sector.

The self-similar structures of LOD 1, LOD 2, LOD 3, ... each nest the structure of the next LOD. When an upper limit of the self-similar range is reached, the structure on the upper bordering LOD, i.e. the last and smallest nested structure, differs from the structures embedding it in that this last and smallest nested structure has no more nesting potential. This also means it cannot generate any further Δt_{depth}, since the generation of Δt_{depth} is achieved through nesting. The structure on the last nesting level of a self-similar structure can be denoted "non-temporal" since it is indivisible and cannot generate Δt_{depth}. Δt_{depth}, though, is a prerequisite for recognising Δt_{length}. Structures of the B-series cannot be recognised without nesting: in order to define incompatible intervals, which comprise Δt_{length}, one must first define and choose, by means of Δt_{depth}, a nesting level or LOD on which these intervals may be arranged.

The last and smallest structure of a self-similar nesting is indivisible, since it has no nesting potential. It is the *prime*.

> *Definition of the prime*. In a nested self-similar structure of the B-series, the most deeply nested structure which has no nesting potential is called the *prime*. As a result of its lack of nesting potential, the prime cannot generate Δt_{depth}. It is indivisible in the Bergsonian sense. Ontologically, it has some, but not all, of the properties of McTaggart's B-series and of the V-series. The prime exhibits extension in Δt_{length} and exists independent of the observer. These are properties of B-series elements. But B-series elements are also defined by their nesting potential, which enables them to generate Δt_{depth}. The prime cannot do this, as it has no nesting potential. Without this potential, no further differentiation within the prime interval can be made. This indivisibility in the Bergsonian sense is also a property of the V-series. But V-series elements have other properties the prime does not share: non-temporality (i.e. implying no duration) and inaccessibility (i.e. via the B-series). The prime is accessible and deducible via the nesting cascades of the B-series. The V-series is not identical with Plato's world of ideas: Plato's ideas are eternal, primes and V-series

elements are non-temporal.[75] Non-temporality denotes a status of inability to generate Δt_{depth}. Since Δt_{depth} is a prerequisite for the possible existence of Δt_{length}, $\Delta t_{depth} = 0$ suffices as a definition of non-temporality.

For natural fractals with an upper and a lower limit to scale-invariance, LOD ∞ will not be reached. In Figure 21, the prime is located on LOD 4.

3.4 Time-condensation as *insight*

Direct access to the prime of a fractal may be possible via the scale-invariant structure of the B-series by means of condensation. This suspicion is backed by the following consideration: Since the same structure exists on all LODs of the B-series, the structure of the prime may be defined as a constant, in order to put all LODs into a relation to each other. Let us assume that, within a nesting of Koch curve structures of the B-series, the structure of the prime on LOD 1 corresponds to Husserl's present, which is extended through retentional nestings and experienced by the subject. This structure is temporal (i.e. it implies duration), that is, it occupies a certain interval Δt_{length} of a certain LOD of the B-series. Figure 21 shows the structure of the Koch curve occupying the entire interval of LOD 1. Taking the basic structure of the Koch curve as a constant, this interval corresponds to Δt_{length} on LOD 2, Δt_{length} on LOD 3, and so on. At the same time, Δt_{length} contracts with every additional nesting. On LOD ∞, Δt_{length} approaches 0.

The structure on the last LOD exhibits properties of both the B-series and the V-series. On the one hand, indivisible structures are not elements of the B-series, since the B-series comprises elements with temporal extension in Δt_{length} and nesting potential. On the other hand, these structures on the last LODs are equal to their corresponding B-series structures: they are, in a sense, derivatives of the latter. The primes "live" on the last LOD and they are essentially non-temporal (i.e. they do not imply duration). Non-temporality is a property of V-series elements, which comprise non-temporal, indivisible, unmodifiable, ideal structures.

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[75] Plato considers time as 'the moving image of eternity' and the forms as eternal. For the fractal concept of time, non-temporality is defined as $\Delta t_{depth} = 0$.

Despite the differences between the V-series and Plato's world of ideas (cf. 3.4 below), it is of advantage, for reasons of transparency, to illustrate the internal relations between A-, B- and V-series with the help of the Platonic Triangle: Each of the three realms of ideas, worldly things and subject corresponds to (at least) one time series. Inaccessible to us, the V-series correlates to the realm of ideas. The B-series, whose structures are extended and exist independent of any observer, describes the temporal order of the realm of worldly things and therefore correlates to that realm. The A-series, which is observer-dependent, i.e. which is based on indexicals such as past, present and future, describes the temporal relations of the indexical realm of the subject and therefore correlates to that realm.

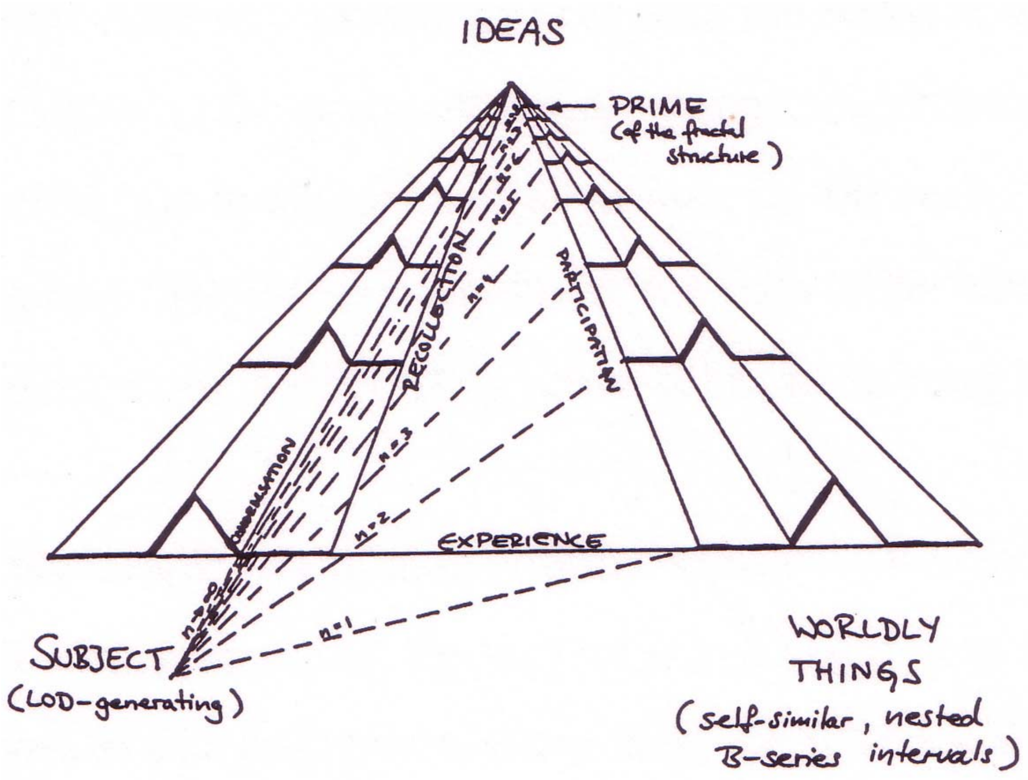

Figure 22

Condensation - the property introduced at the beginning of this chapter - describes the relation between the A-series and the prime, first via the B-series, which comprises self-similar structures with extension in Δt_{depth} and Δt_{length}. Δt_{length} contracts further and further with every nesting, until there is direct contact between the A-series and the prime.

In terms of Plato's Triangle, the realm of the subject can make direct contact with the realm of ideas, indirectly via the prime, through condensation: the first step is taken through experience via the realm of worldly things, and the second step is taken, on the last LOD, via the prime.

As Figure 22 shows, the condensation link correlates, in the fractal model, with a recollection link analogous to the anamnesis link in Plato's Triangle. The terms 'recollection' and 'anamnesis' are not identical, though. 'Recollection' does not imply characteristics such as the cleansing of the soul from the encrustation of bodily cares and interests, as the Platonic term 'anamnesis' does. Recollection in the fractal model serves as a step by step approach of the subject (via the Now of the A-series) towards the realm of ideas. This occurs through experience via the realm of worldly things (whose internal relations are described by the B-series). Nature's participation in the ideas, relates, in the fractal model, exclusively to a structural congruence: the prime of the fractal exhibits characteristics of both the realm of worldly things and the realm of ideas. In the realm of worldly things, the prime occupies a certain Δt_{depth} and Δt_{length}. In the realm of the prime, i.e. the realm between that of worldly things and that of ideas, which corresponds to the last nesting, the prime cannot generate any further Δt_{depth}, since it has no nesting potential.

Through favourable positioning within congruent nestings, i.e. in the prime of the self-similar structure, the subject is in a position to recognize the V-series properties of this prime. In that moment, the B-series in its function as a connecting link between A-series and V-series properties is released: the contact between subject and prime is non-temporal (or, in Penrose's terms, non-algorithmic).

Condensation can only take place under the following conditions:

1. A nesting of B-series must be recognized or generated.

2. This nesting must exhibit scale-invariance, preferably in the form of structures of the prime recurring on all levels available.

3. The scale-invariance must be other than statistical. A statistical scale-invariance does not suffice, since a statistic determination always rests on a large amount of data and therefore implies divisible extension: it is not possible to define a prime on the last LOD for statistical scale-invariance.

4. The subject's position must be an extended present in Husserl's sense, which comprises the basic structure of the prime of the fractal in question.

If these conditions are met, it is possible to catch a glimpse of the larger structure embedding the present, by, metaphorically speaking, looking into the opposite direction to the V-series: there, the prime of the present is embedded in larger intervals of the same basic structure. This glimpse is rendered possible through the congruence of the prime structure on all LODs. This intuition which comprises the past as well as the present and the future, corresponds to Penrose's concept of *insight*[76]. In the fractal model, this moment of intuition corresponds to the καιρος , the key moment in which the future may be influenced.

Most natural fractals exhibit an upper and a lower boundary, so that the nesting level ∞ will practically not be accessible. But even a limited scale-invariant nesting suffices to allow us to catch a glimpse of the structure embedding our particular present. Such scale-invariant nestings may be generated artificially, through fractal musical compositions, for example[77]. Other scale-invariant nestings are probably structured through and by ourselves. As human beings with more or less extended bodies, we are subject to temporal structures steered by our metabolism, hormone regulation, and other bodily functions. Many of these processes exhibit scale-invariances[78].

The virtual presence of temporal structures in non-temporal ones (primes) is reminiscent of Leibniz' concept of the monad. The monad mirrors the universe and, in a virtual sense, already comprises a temporal structure:

> "As 'repraesentatio mundi', the world is present in it not just spatially but also temporally. Just as every spatially formed organism finds its corresponding representation in the monad, so does every temporal series."[79]

Leibniz does not attribute any reality to time (except for the 'momentaneum') - it is an ordering principle which relates only to the world of phenomena: time is the order in which what is incompatible is real[80]:

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[76] Penrose's explanation of *insight* varies considerably from this fractal approach. Cf. Penrose 1989.
[77] Such a fractal concert (composed by Peter Neubäcker - see bibliography) was performed at the Annual Symposion of the Munich Chaos Group on 13.11.1993.
[78] Cf. Olsen et al 1987.
[79] Rudolph 1989, p. 121 (my translation).
[80] Cf. Böhme 1974, p.251.

> "Even what is mutually incompatible is still reflected by the monad in a unified way. This reflection does not occur in accordance to the ordering principle of time; the relation of the monad to the unfolded time series, i.e. to the 'empirical time series', corresponds to the relation of the unity of time to the continuous succession of its modes."[81]

The most significant difference between the monad / empirical time relation and the prime / B-series relation lies in the status of these concepts. The difference between Leibniz' concepts of monad and time is analogous to the difference between substance and phenomenon[82]. In contrast, the fractal model allows an arbitrarily close approach of a B-series structure towards its prime. Even if an approach such as this is executed only in part, it allows the arbitrary dilation of a present nested in a self-similar structure by making nesting LODs accessible through condensation.

This paper can touch only in a rudimentary way on the question of how scale-invariant structures may be generated. Questions concerning a possible exploitation of condensation in scale-invariant structures must also be debated elsewhere. Here, it shall suffice to present a way of seeing the world. Whether or not interference through time condensation is sensible or ethically justifiable would have to be discussed and decided on before possible application.

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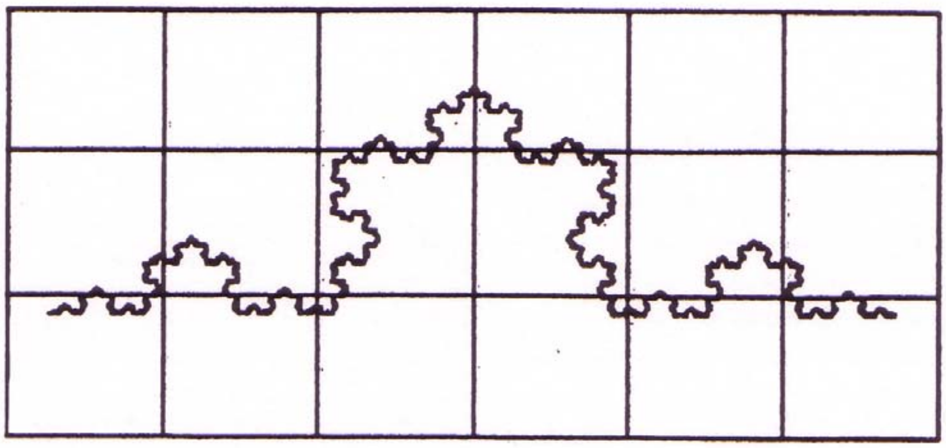
[81] Rudolph 1989, pp. 121f. (my translation).
[82] Cf. Rudolph 1989, p.122.

4. Appendices

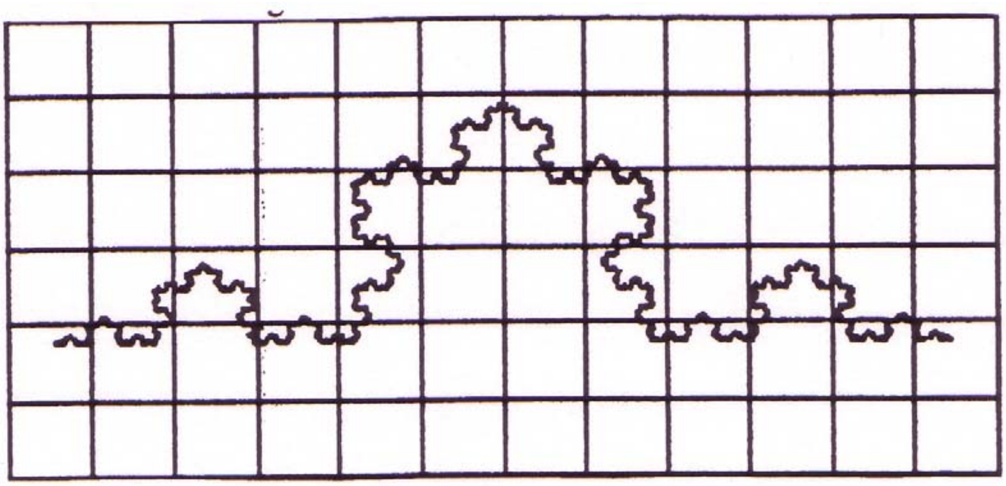
4.1 Koch curve and Menger sponge

If neither the scaling factor nor the initiator for the generation is known, the fractal dimension can be determined, according to Barnsley[83], by means of the Box Counting Method. This has been done for the Koch curve below. With every refinement of the grid, one gets closer to the limit 1.2618..., the fractal dimension of the Koch curve.

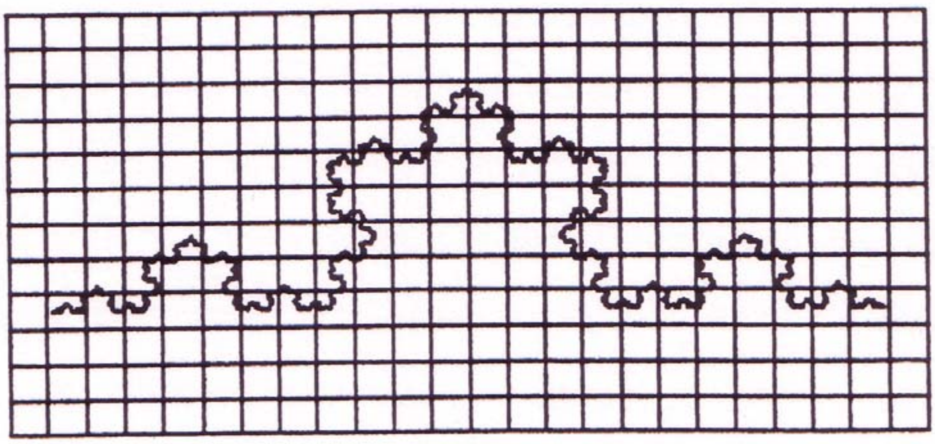
For squares of side length 1/6 : ln 14 / ln 6 = 1.4738...



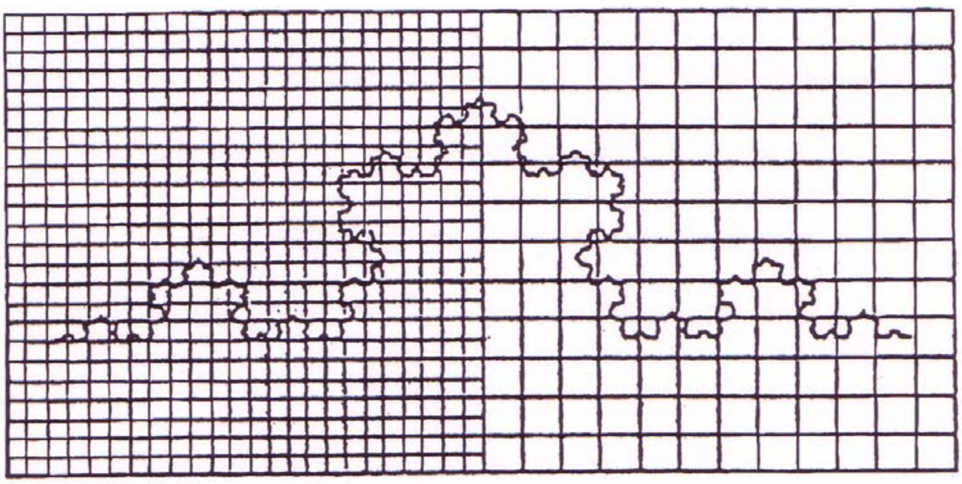
For squares of side length 1/12: ln 26 / ln 12 = 1.3111...



For squares of side length 1/24: ln 58 / ln 24 = 1.2776...



For squares of side length 1/48: ln 134 / ln 48 = 1.2651...



If the scaling factor and the number of identical parts are known (and therefore also the generating rule), the fractal dimension is equal to the self-similarity dimension, which is easily determined:

$$d = \frac{\ln(\text{number of parts})}{\ln(\text{scaling factor})} = \frac{\ln 4}{\ln 3} = 1.2618...$$

While the fractal dimension of the Cantor dust lies between 0 and 1, that of the Koch curve lies between 1 and 2. The Menger sponge provides an illustrative example of a fractal of a dimension between 2 and 3. Again, the fractal dimension may be determined by means of Barnsley's Box Counting Method, as has been done in the example above, or simply by

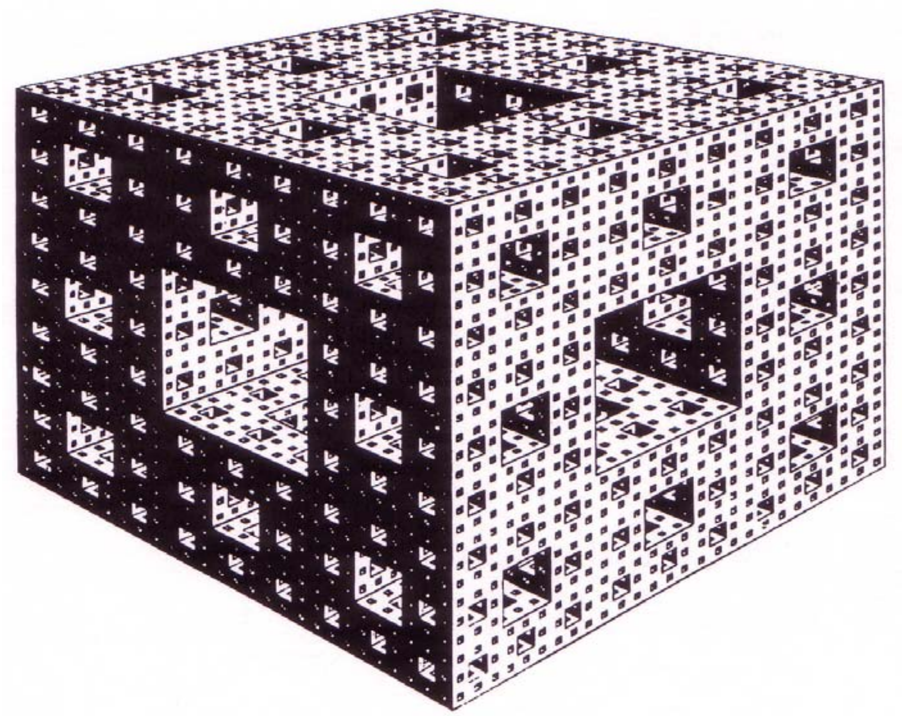
[83] Barnsley 1988.

figuring out the quotient of the number of parts to the scaling factor. The latter method rests on the assumption that both quantities, the scaling factor and the number of identical parts, are known. Let us assume then, for the next example, that the generating rules are known: the Menger sponge is generated by dividing a cube's sides into nine congruent squares and by then taking out cubes of side length 1/3 (of the initiator) of the middle square from each side as well as one cube of side length 1/3 (of the initiator) out of the middle of the main cube. The remaining cube now comprises 20 congruent cubes of side length 1/3. In the next step, the whole operation is repeated for every remaining cube. The iteration may go on ad infinitum and produce the Menger sponge (see Figure 23):

For cubes of side length 1/3, i.e. $n = 1$: $\ln 20 / \ln 3 = 2.7268...$

For cubes of side length 1/9, i.e. $n = 2$: $\ln 20^2 / \ln 3^2 = 2.7268...$

For cubes of side length 1/27, i.e. $n = 3$: $\ln 20^3 / \ln 3^3 = 2.7268...$



The relation of the individual parts to the scaling factor remains constant, therefore, the fractal dimension of the Menger sponge equals 2.7268... .

Figure 23

4.2 Statistical scale-invariance in the distribution of pauses

As reported in Chapter 2.1, I have searched for scale-invariance in the distribution of pauses in phonograms[84] of a text retold by French and German native speakers. Figure 24 shows an example of such a phongram. The amplitudes represent differences in volume. There are, as shown in Figure 24, long intervals without change in volume. If one projects a phonogram onto a Cartesian co-ordination system, the amplitudes on the y axis represent the volume of speech, plotted against time on the x axis (one square on the x axis corresponds to 0.08 seconds). In this search for scale-invariance, semantic aspects remained unconsidered and pauses were defined as arbitrarily large or small intervals for which y = 0.

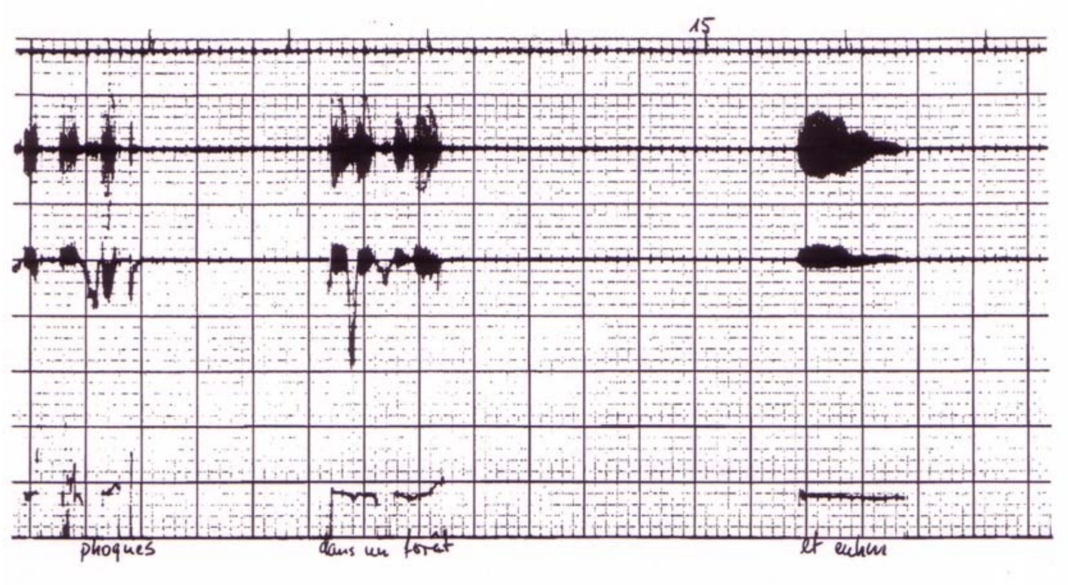


Figure 24

LODs were, for each case, defined by the minimum length of an interval with y = 0. LOD 1, for example defines pauses as intervals with y = 0, which last at least 2.25 seconds (corresponding to 27 squares). LOD 3 defines pauses as intervals with y = 0 of a minimum length of 0.25 seconds (corresponding to 3 squares), and so on. The scaling factor relating one LOD to the next was, in each case, 3.

LOD 0: pauses ≥ 81 squares : 6.75 seconds.
LOD 1: pauses ≥ 27 squares : 2.25 seconds.
LOD 2: pauses ≥ 9 squares : 0.75 seconds.
LOD 3: pauses ≥ 3 squares : 0.25 seconds.
LOD 4: pauses ≥ 2 squares : 0.16 seconds.

[84] Data provided by Dechert and Raupach. Cf. Dechert /Raupach 1980.

LOD 5: pauses ≥ 1 squares : 0.08 seconds.

The pause intervals were, for each LOD, traced onto transparencies. The result may be presented in various forms. Figure 25 shows the pause intervals for FM1, a French native speaker, shown in the form of a bifurcation cascade, and Figure 26 shows the same data in the form of linear parallel arrangements. Both forms expose the nesting of the various LODs.

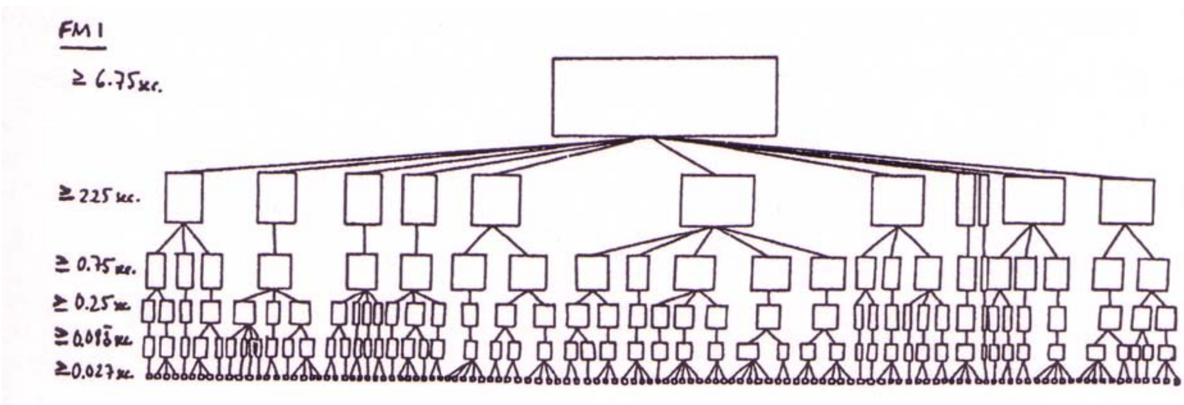


Figure 25

The linear form is reminiscent of the Cantor dust, but the distribution of pauses does not exhibit the symmetrical properties of the Cantor dust. It was possible, though, to trace a statistical self-similarity for several scales. The distribution of pauses plotted on the log-log scale in Figure 9 (Chapter 2.2) exhibits a scale-invarinace for the scaling factor 3. It was not possible to find any correlations between the self-similarity and properties such as the native language or the gender of the speaker.

Figure 26

About 80% of the data I analysed showed a peculiar aberration for the scaling factor 3, on LOD 3, for which a pause is defined as an interval with y = 0 for a minimum length of 0.25 seconds.

I shall refrain from speculation concerning possible correlations, as this is irrelevant for our purposes. A thorough study of the distribution of pauses requires the analysis of a vast amount of data. For our purposes, it shall suffice to present a LOD-independent method of analysing pausology data, which provides less arbitrary and more encompassing time series analyses. Within the context of this paper, the pausology examples serve only one purpose, namely to illustrate a method for the LOD-independent presentation of a time series. Figures 27 - 30 show examples of the peculiar aberration for the scaling factor 3, on LOD 3.

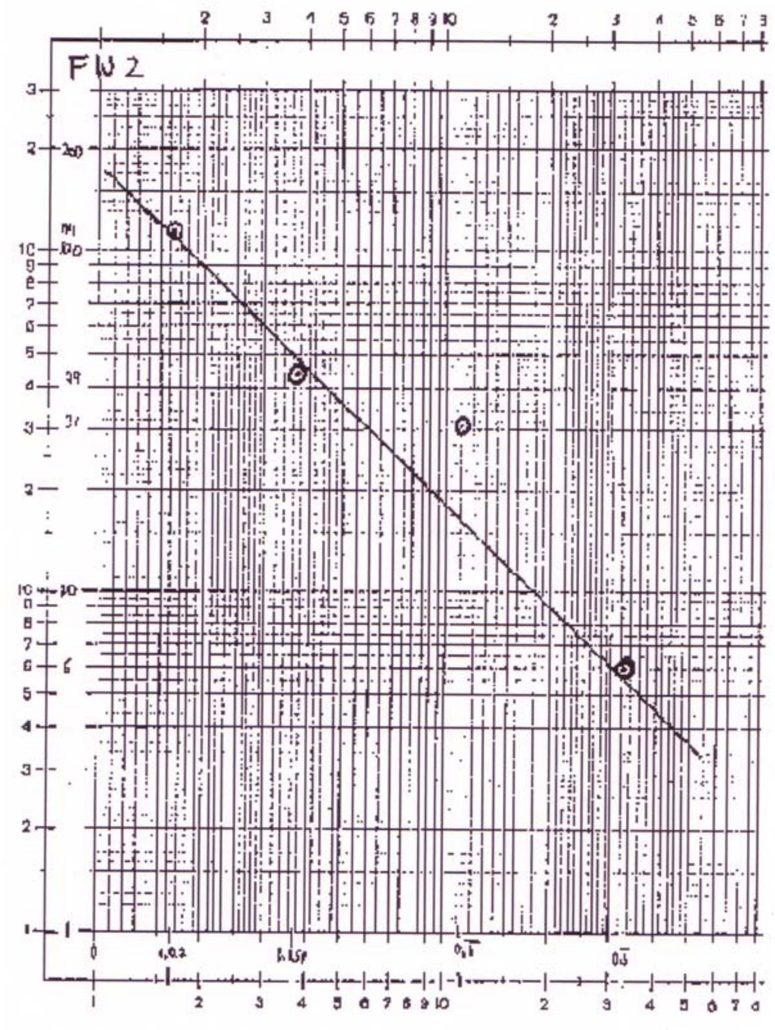


Figure 27

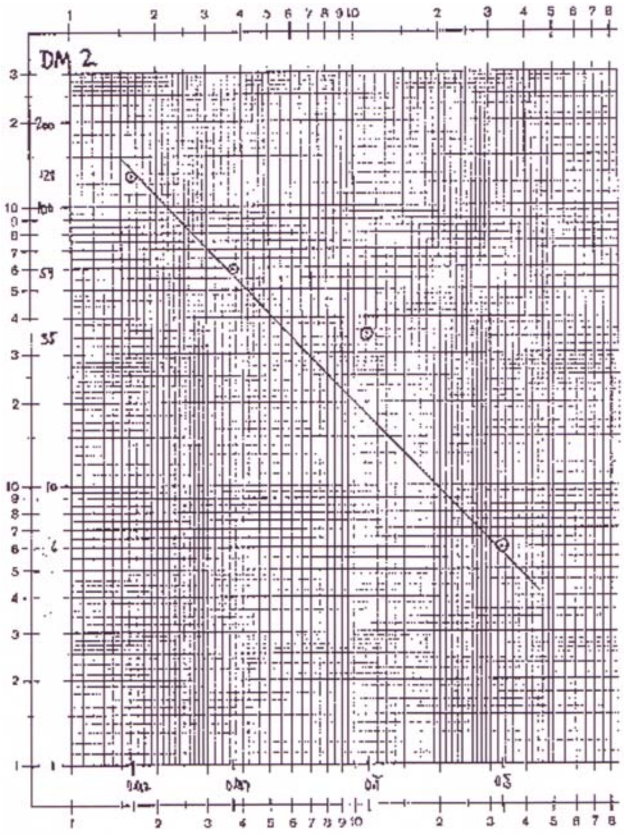


Figure 28

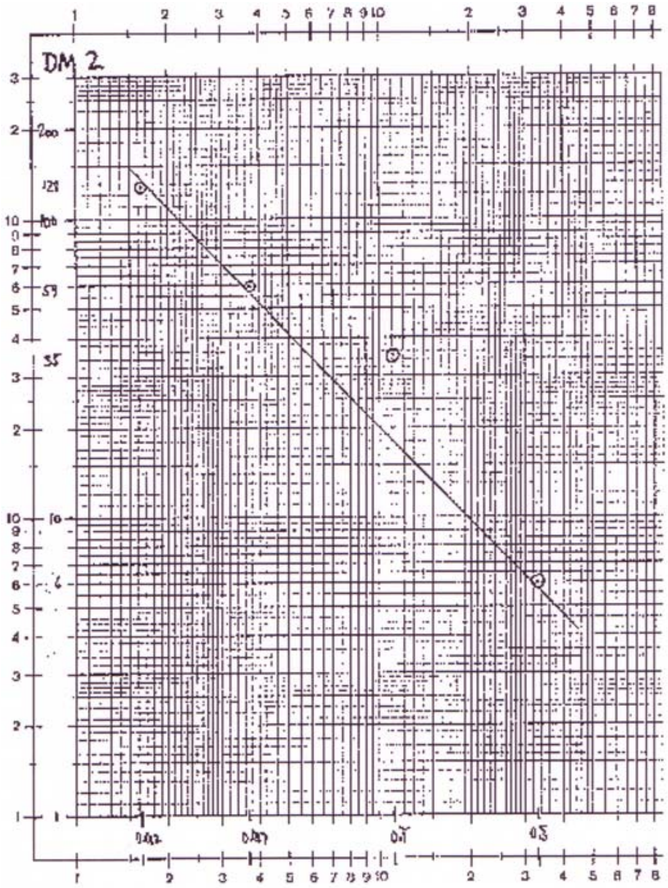


Figure 29

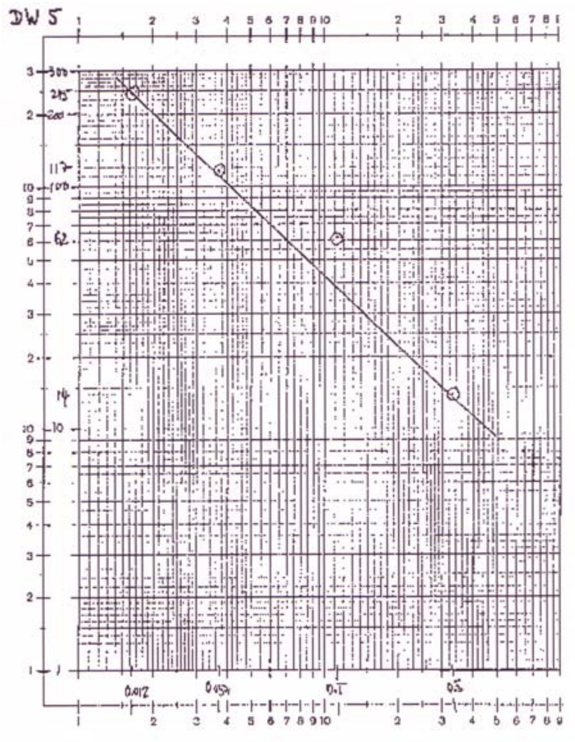


Figure 30

4.3 Dendrochronological data

The dendrochronological analysis introduced in Chapter 3 was conducted by the following method. The data of *1000 year oak chronology*[85] were searched, for selected intervals within the period 1600-1960, for scale-invariant structures (see Figure 31).

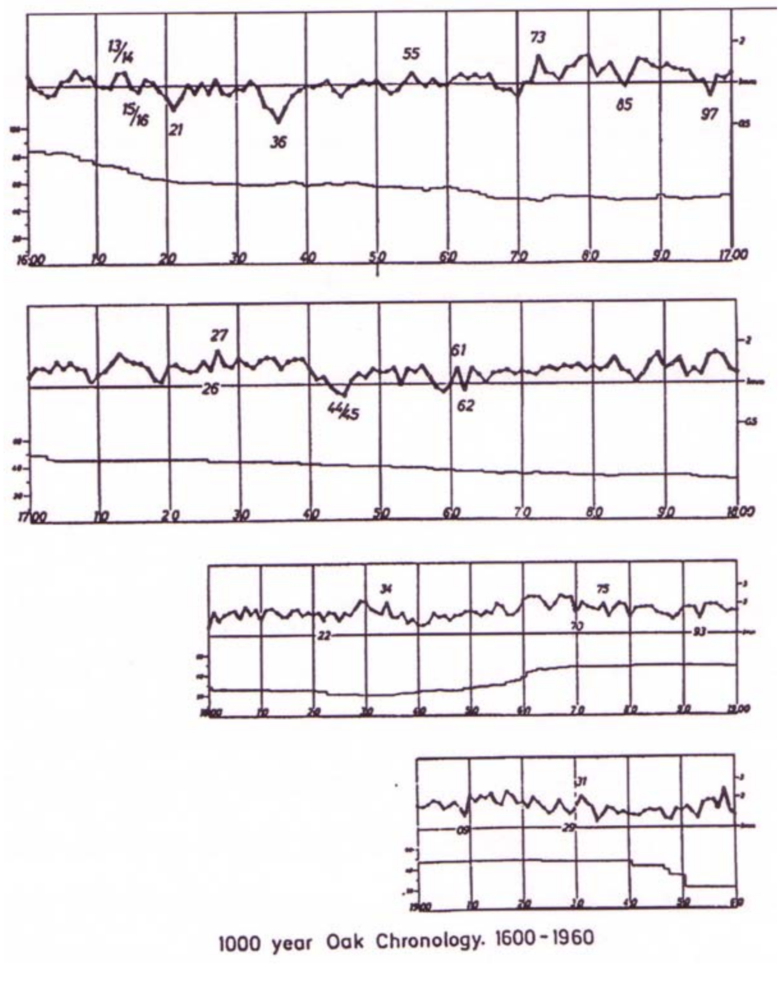


Figure 31[86]

In order to search for

[85] Cf. Fletcher 1978.
[86] from: Fletcher 1978, p. 31.

scale-invariance in the change in ring width of European oaks, choosing different years as starting points, variations were recorded for several delays, e.g., the variations registered every 3 years, and presented graphically (every interval between dots on the x axis represents a 3-year interval).

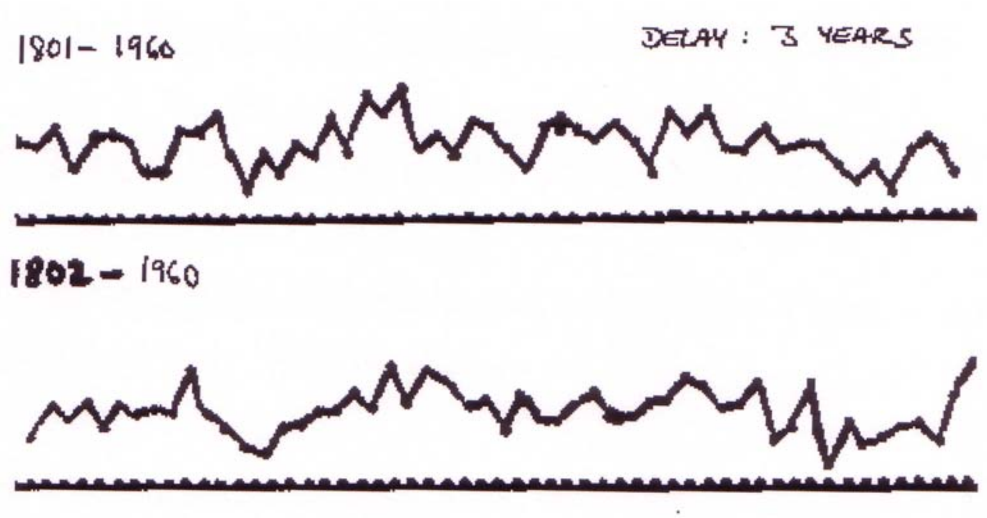


Figure 32

The curve in Figure 32 shows the variations in the ring width as registered every 3 years. Variations in the ring width were recorded also for delays of 5, 7, 9, 10, 11, 13, 17, and 27 years. The curves which resulted from this were searched for congruence by means of superimposing the transparencies showing the curves for each delay. Two of the self-similar nestings were presented in Chapter 3 (Figures 17 and 18). For our purposes, it shall suffice to exemplify the fact that scale-invariances are a frequent symmetrical property to be found in time series. A more encompassing study of scale-invariance in dendrochronological data, which might reveal correlations with climatic changes and the like, would have to be conducted elsewhere.

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